Oil Prices, Inflation Expectations, and Bond Risk Premiums

Haibo Jiang*

University of Quebec at Montreal

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*School of Management, University of Quebec at Montreal, Montreal, Quebec H2X 3X2, Canada. Email: jiang.haibo@uqam.ca, Tel: (514) 987-3000 ext 5521. This paper is based on chapter 2 of my doctoral dissertation at University of British Columbia. I am grateful to my advisor Lorenzo Garlappi and dissertation committee members Paul Beaudry, Murray Carlson, Jack Favilukis, Carolin Pflueger, and Georgios Skoulakis for invaluable guidance and encouragement. I thank Alexander David, Adlai Fisher, Ron Giammarino, Andrea Heuson, Steffen Hitzemann, Yan Ji, Zeigham Khokher, Marcelo Ochoa, Eduardo Schwartz, seminar participants at the 8th International Symposium on Environment and Energy Fiannce Issues (ISEFI-2022), the 2019 China International Conference in Finance (CICF), the 2018 European Finance Association Annual Meeting, the 2018 Financial Management Association Annual Meeting, the 2018 Midwest Finance Association Annual Meeting, the 2015 Northern Finance Association Annual Meeting PhD sessions, Stevens Institute of Technology, UBC Vancouver School of Economics, and UBC Sauder School of Business for helpful suggestions and comments. I also thank Jonathan Scher for editorial assistance. I gratefully acknowledge financial support from the SSHRC and the Canadian Securities Institute.

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Abstract

Using oil supply, global demand, and oil-specific demand shocks, estimated from a structural VAR model of oil price changes, this paper first provides new empirical evidence: (i) the dissecting of oil price changes improves CPI and core CPI inflation forecasts, and (ii) the oil price changes driven by global demand shocks predict negative real bond risk premium and positive inflation risk premium. Since these two effects offset each other, we observe insignificant effect on the bond risk premium. A two-sector New Keynesian model further shows theoretically that real bond yield, breakeven inflation, and nominal bond yield respond differently to oil supply and demand shocks. The model helps to explain (i) the muted impact of 2000s oil crisis on expected inflation and (ii) the comovement of expected inflation and the oil price surge that resulted from Russia's invasion of Ukraine.

JEL Classification Codes: E31; E43; E44; G12; G14; Q41; Q43

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1 Introduction

Oil is the most important commodity in the economy as it is both consumed by households to meet their daily energy needs and used by firms as energy input to produce a wide range of goods and services.[1] Rising oil prices directly affects the Consumer Price Index (CPI) because gasoline accounts for about 4-6% of the overall CPI and indirectly passes through to the prices of most goods and services because oil is required to produce or deliver these goods and services. Central banks naturally pay close attention to oil prices, especially for forecasting inflation. Existing research shows that oil prices have little or no predictive ability for short-term CPI inflation (see Pasaogullari and Waiwood (2014) among others). However, given that recent academic literature has discussed the potential differential effects of demand and supply shocks associated with oil price fluctuations (Kilian, 2009; Caldara, Cavallo, and Iacoviello, 2019; Baumeister and Hamilton, 2019), it is important to study whether oil supply and demand shocks can help to predict CPI inflation.

In addition, it is also important to study the extent to which fluctuations in oil prices are related to the prices and expected returns of financial securities. Indeed, many studies, beginning with Chen, Roll, and Ross (1986), have investigated the role of oil prices in stock returns.[2] However, how oil price changes affect bond returns and inflation component of nominal bond yields remains an open question. Few papers have studied the impact of oil prices on bond returns and yields.[3] Compared with stocks, bonds are supposed to be more directly affected by fluctuations in oil prices through the expected inflation component in

[1]Global expenditures on petroleum account for about 4.5% of the world GDP. The average U.S. household spends about 4% of pre-tax income on gasoline for day-to-day transportation, and about 40% of industrial energy consumption is accounted for by oil, according to data in 2013 from the U.S. Energy Information Administration.

[2]Driesprong, Jacobsen, and Maat (2008), Kilian and Park (2009), Chiang, Hughen, and Sagi (2014), Jiang, Skoulakis, and Xue (2018), among others, find that oil prices impact stock returns.

[3]The only two exceptions are studies by Kang, Ratti, and Yoon (2014) and Baker and Routledge (2017). Kang, Ratti, and Yoon (2014) show that U.S. Treasury bond returns deflated by the U.S. CPI are negatively associated with oil price shocks driven by global aggregate demand for all industrial commodities. Baker and Routledge (2017) document that monthly excess returns on nominal U.S. Treasury bonds are higher when the slope of NYMEX WTI crude oil futures curve is negative.

nominal bond yields. Astonishingly, prior literature finds little predictive power of oil price changes on bond excess returns. This is especially surprising since intuition and casual empirical observations suggest an important link between the two. For instance, the yield on the U.S. 10-year Treasury bond rose 8 basis points, along with a sharp rise of 4.6% in the WTI crude oil price, on headlines related to the sanctions of Russian oil and the war in Ukraine on March 8, 2022. Furthermore, the relationship between oil prices and bond yields varies over time, which seems particularly puzzling in the period of 2000s energy crisis. As shown in Figure 1, both yields on U.S. 10-year Treasury securities and Treasury Inflation-Protected Securities (TIPS) remained stable at the low level when the crude oil prices experienced a persistent run-up from about US$30 per barrel in 2003 to US$147 per barrel in 2008. The objective of this paper is to provide empirical and theoretical analyses on the relations between oil prices, inflation expectations, and bond yields and returns.

To fully understand the link between oil prices, inflation, and bond returns, it is crucial to further decompose oil price changes because not all oil shocks are the same (Kilian, 2009). An oil price hike could be bad news for the economy if driven by a scarce oil supply, or good news if driven by a strong demand for oil. Following Jiang, Skoulakis, and Xue (2018), I use a structural VAR approach to decompose oil price changes into oil supply, global demand, and oil-specific demand shocks, which are used as regressors to predict inflation and to explain and predict excess returns on Treasury bonds, TIPS, and breakeven inflation.

Following the methods proposed by Pasaogullari and Waiwood (2014), I use the root mean squared error (RMSE) to compare the accuracy of various models of predicting one-year-ahead CPI inflation and core CPI inflation. Results suggest that oil supply, global demand, and oil-specific demand shocks have predictive value for forecasting CPI inflation for the periods of 1986Q1–1999Q4, 2010Q1–2020Q4, and 1986Q1–2020Q4 and for forecasting core CPI inflation for the periods of 2010Q1–2020Q4, and 1986Q1–2020Q4. These results show the relative importance of oil supply and demands shocks on inflation forecasting.

This paper also provides empirical evidence that the oil price changes are negatively associated with excess returns on U.S. 10-year Treasury bonds and breakeven inflation (the difference in excess returns between nominal government bonds and inflation-indexed bonds of equivalent maturity) and explain an additional 9% of the variation in excess returns.[4] In addition, oil price changes predict excess returns on breakeven inflation with a positive slope, but fail to predict nominal bond excess returns. This finding is counter intuitive, especially given the strong predictive relationship between oil price changes and equity risk premium (Driesprong, Jacobsen, and Maat, 2008). Regression results show that oil supply shocks weakly predict only excess returns on breakeven inflation with a positive slope. More importantly, global demand shocks predict negative real bond risk premium and positive inflation risk premium. Since these two effects offset each other, we observe insignificant effect on the nominal bond risk premium.

For the theoretical analysis, I build a two-sector New Keynesian model of an economy where oil and core goods are produced in an oil sector and a core sector, respectively. A critical feature of the model is that oil is included in households' utility function (Hitzemann, 2016; Ready, 2017) and used as an input in the production of core goods (Blanchard and Galí, 2010). In addition, household consumption of oil is assumed to be complementary to the consumption of core goods.[5] In equilibrium, the rise of oil price could be driven by either negative productivity shocks in the oil sector or positive productivity shocks in the core sector. The former shock is viewed as the negative oil *supply* shock, and the latter acts as the positive oil *demand* shock.

The model has three predictions. First, real bond yields respond positively to increases

[4]In addition, oil price changes can also explain and predict excess returns on inflation swaps, which are market-based measures of breakeven inflation. Inflation swaps data have been used by Haubrich, Pennacchi, and Ritchken (2012), Fleckenstein, Longstaff, and Lustig (2013, 2014), and others.

[5]The elasticity of substitution between oil and core goods is less than one, supported by empirical findings by Ready (2017).

in oil prices, irrespective of the type of oil shocks. Second, an increase in oil price raises breakeven inflation after a negative oil supply shock, and lowers breakeven inflation after a positive productivity shock in the economy. Breakeven inflation plays an important role in the joint dynamics of oil prices and bond yields. Third, the conventional wisdom that oil price hikes are associated with increases in CPI inflation and nominal bond yields is true only if the rise of oil price is driven by a negative oil supply shock. In contrast, an oil price hike lowers breakeven inflation and nominal interest rates if the oil price hike is driven by strong demand for oil.

The predictions of the model help to explain (i) the muted impact of 2000s oil crisis on expected inflation and (ii) the comovement of expected inflation and the oil price surge that resulted from Russia's invasion of Ukraine in 2022. For the 2000s energy crisis, the strong demand for oil from emerging economies is considered as one of the major drivers of the long-run oil price hikes, while the slow growth in oil production is viewed as a contributing factor too. According to the model, for the increases in oil prices driven by the strong demand in the economy, there is downward pressure on the breakeven inflation; for the increases in oil prices driven by the slow growth in oil output, there is upward pressure on the breakeven inflation. These two effects may explain why the breakeven inflation and nominal bond yields remained stable in 2000s. However, for the oil price surge that resulted from Russia's invasion of Ukraine since February 2022, the sanctions to Russian energy sector is the main cause, which is viewed as a negative shock to the world oil supply. As the model predicts, breakeven inflation rises in response to the negative oil supply shock. We thus observe the spike in breakeven inflation and nominal bond yields along with the surge of oil prices, as the aforementioned example of events on March 8, 2022.

This paper is related to a growing literature on studying determinants of nominal and real bond yield curves. Previous papers studying real rates, inflation expectations, and risk premia use latent factor term structure models (Ang, Bekaert, and Wei, 2008;

Chernov and Mueller, 2012; Haubrich, Pennacchi, and Ritchken, 2012) and New Keynesian macro models (Kung, 2015; Hsu, Li, and Palomino, 2014). However, oil prices have not been considered in this literature. In this paper, the price of oil is treated as an explicit macroeconomic risk factor. In particular, this paper extends the standard New Keynesian model by adding an oil sector and incorporating the dual uses of oil by households and firms and further examines macroeconomic linkages among real and nominal bond yields, breakeven inflation, and oil supply and demand shocks.

This paper is also related to several empirical papers that document the connection between oil spot or futures prices and U.S. Treasury bond returns. Kang, Ratti, and Yoon (2014) show that U.S. Treasury bond returns deflated by the U.S. CPI are negatively associated with oil price shocks driven by global aggregate demand for all industrial commodities. Baker and Routledge (2017) document that monthly excess returns on nominal U.S. Treasury bonds are higher when the slope of NYMEX WTI crude oil futures curve is negative. Surprisingly, few papers study the impact of oil prices on long-term breakeven inflation, although numerous studies examine the effect of oil prices on short-run core inflation and total inflation, as reviewed in detail by Clark and Terry (2010). Celasun, Mihet, and Ratnovski (2012) find that oil futures price shocks have a statistically significant impact on long-term breakeven inflation. In fact, both real rates and breakeven inflation are important in understanding nominal bond yields (Duffee, 2014; Pflueger and Viceira, 2016). This paper is the first to examine not only nominal bond yields as a whole, but also real bond yields and breakeven inflation separately in relation to oil prices.

Last, several recent papers have studied the impact of oil price shocks on equity returns. Driesprong, Jacobsen, and Maat (2008) find that increases in oil prices predict low future stock returns. Jiang, Skoulakis, and Xue (2018) show that oil supply shocks and oil-specific demand shocks predict MSCI country index equity returns with a negative slope and global demand shocks predict equity returns with a positive slope. Kilian and Park (2009) show that oil supply and demand shocks jointly explain 22% of the long-run variation in U.S. real stock returns. Chiang, Hughen, and Sagi (2014) demonstrate that oil risk factors explain the returns of non-oil portfolios. As a complement to the literature on the relationship between oil price and equity returns, this paper focuses on the impact of fluctuations in oil prices on bond returns and breakeven inflation.

This paper makes three contributions to the literature. First, this paper presents novel empirical evidence that oil supply, global demand, and oil-specific demand shocks have predictive power for CPI inflation and excess returns on TIPS and breakeven inflation. Empirical tests using data on TIPS provide richer tests on different behaviors of components of nominal yields, with respect to oil shocks, than those using data solely on nominal bonds. Second, using the oil supply shocks and global demand shocks decomposed from oil price changes, this paper shows that global demand shocks predict negative real bond risk premium and positive inflation risk premium. Since the two effects offset each other, we observe insignificant effect of oil price changes driven by global demand shocks on the nominal bond risk premium. Third, this paper builds a two-sector New Keynesian model to examine the theoretical relationships between nominal and real yields, breakeven inflation, and oil supply and demand shocks. The model offers novel predictions and further highlights key economic transmission channels through which oil price shocks affect bond markets.

The remainder of this paper is organized as follows. Section 2 describes the data and presents empirical results. A two-sector New Keynesian model is presented in Section 3. Section 4 discusses model calibration and oil market implications. Section 5 provides theoretical analysis, replicates empirical regressions using simulated data, and highlights the importance of disentangling oil and supply shocks. Section 6 uses the model's predictions to reconcile the intriguing relationships between oil and bond yields and breakeven inflation in the 2000s energy crisis and the Russian oil supply shock of 2022. In Section 7, I offer some concluding remarks.

2 Empirical results

This section first describes data and then presents empirical evidence on the predictive ability of oil supply and demand shocks for CPI inflation and core CPI inflation. In addition, tests are conducted for the explanatory and forecasting power of crude oil price changes for excess returns on nominal bonds, real bonds, and breakeven inflation. Furthermore, empirical tests are conducted using oil supply shocks, global demand shocks, and oil-specific demand shocks.

2.1 Data

Three-month interest rates are used to construct excess returns for U.S. 10-year Treasury bonds and U.S. 10-year inflation-indexed bonds called Treasury Inflation Protected Securities (TIPS). Excess returns on breakeven inflation rates are defined as the difference in excess returns between nominal bonds and TIPS. Data on U.S. 10-year nominal Treasury bond yields, liquidity-adjusted U.S. 10-year TIPS yields, and liquidity-adjusted breakeven inflation from June 1999 to December 2014 are from Pflueger and Viceira (2016).[6] TIPS yields and breakeven inflation are adjusted for the liquidity risk present in the TIPS market (for details, see Pflueger and Viceira (2016)).

Crude oil spot prices are based on U.S. refiner acquisition cost of imported crude oil, and the data since January 1974 are obtained from the U.S. Energy Information Administration (EIA). Historical inflation data on the seasonally-adjusted Consumer Price Index and subindexes from January 1947 are obtained from the U.S. Bureau of Labor Statistics. Data on inflation expectation from University of Michigan's Survey of Consumers and from the Survey of Professional Forecasters are obtained from the websites of the St. Louise Fed and Philadelphia Fed, respectively.

[6] I am very grateful to Carolin Pflueger for providing data constructed in Pflueger and Viceira (2016).

The log growth rates of U.S. refiner acquisition cost of imported crude oil are further decomposed into oil supply shocks, global demand shocks, oil-specific demand shocks. The seminal paper Kilian (2009) proposes using a structural VAR framework to estimate demand and supply shocks in the global crude oil market by decomposing shocks to the real oil prices into oil supply shocks, aggregate demand shocks, and oil-specific demand shocks. Jiang, Skoulakis, and Xue (2018) uses a variant structural VAR framework with stationary input variables and further includes the price elasticities of oil supply and oil demand in the VAR specification to estimate the three shocks, as advocated by Caldara, Cavallo, and Iacoviello (2019) and Baumeister and Hamilton (2019). In particular, the log growth rates of U.S. refiner acquisition cost of imported crude oil are used to estimate the three shocks. The decomposition uses a structural VAR method proposed by Jiang, Skoulakis, and Xue (2018).

2.2 Predictive ability of oil for CPI

Following Pasaogullari and Waiwood (2014), I use the root mean squared error (RMSE) to compare the accuracy of various models of predicting one-year-ahead CPI inflation and core CPI inflation without oil prices, with oil prices, and with oil price shocks estimated in a structural VAR. The one-year-ahead annual CPI inflation (or core CPI inflation) is predicted using the current and four lagged quarterly CPI inflation (or core CPI inflation) (annualized), or with the current and four lagged oil price growth rates, or with the current and four lagged oil supply shocks, global demand shocks, and oil-specific demand shocks.

Figure 2 shows that the regressions with CPI inflation and oil supply shocks, global demand shocks, and oil-specific demand shocks have the lowest RMSE for all the three sub-sample periods and the full sample period; this suggests that oil supply, global demand, and oil-specific demand shocks have predictive value for forecasting CPI inflation. Figure 3 shows that the regressions with core CPI inflation and oil supply shocks, global demand shocks, and oil-specific demand shocks have the lowest RMSE for for forecasting core CPI inflation for the periods of 2010Q1–2020Q4 and 1986Q1–2020Q4.

Inflation expectation is also used in place of quarterly CPI inflation to predict the one-year-ahead CPI inflation. Figures 4 and 5 show the RMSEs of using the median expectation of inflation expectations from University of Michigan's Survey of Consumers (UM) and from the Survey of Professional Forecasters (SPF), respectively. Results show that the three decomposed oil shocks have predictive value for forecasting CPI inflation for all the periods considered, except for the period of 2000Q1–2009Q4.

These results confirm that oil price changes have little or no predictive value for forecasting CPI inflation, as in Pasaogullari and Waiwood (2014), and further demonstrate the relative importance of oil supply and demands shocks in predicting CPI inflation and core CPI inflation.

2.3 Empirical evidence on bond returns using oil price changes

Do crude oil prices have explanatory and incremental forecasting power for bond returns and breakeven inflation? I first use reduced-form regressions to address this question. The independent variable is the log growth rate of crude oil spot prices, denoted by g^{Oil}. I also include two control variables. The first control variable is the term spread, which is a well-known predictor variable for bond returns (Ludvigson and Ng, 2009). Another control variable is CPI less energy inflation, which contains inflation-related information for the breakeven inflation component in the nominal bond yields.

Excess bonds returns refer to one-period buy-and-hold returns in excess of Treasury Bill rate. Excess return of the n-period Treasury bond is defined as $xr^{\$}_{t+1} = ny^{\$}_{n,t} - (n-1)y^{\$}_{n-1,t+1} - y^{\$}_{1,t}$, where $y^{\$}_{n,t}$ is the nominal yield of the n-period at time t and $y^{\$}_{1,t}$ is the rate of the one-period nominal Treasury Bills. Similarly, the liquidity-adjusted log excess return of the n-period inflation-indexed bond is defined as $xr_{t+1}^{TIPS} = ny_{n,t}^{TIPS,adj} - (n-1)y_{n-1,t+1}^{TIPS,adj} - y_{1,t}^{TIPS}$, where $y_{n,t}^{TIPS,adj}$ is the liquidity-adjusted real yield of the n-period TIPS at time t and $y_{1,t}^{TIPS}$ the yield of the one-period real bond.[7] The liquidity-adjusted log excess breakeven return is defined as $xr_{t+1}^{BE} = xr_{t+1}^{\$} - xr_{t+1}^{TIPS}$, representing the log excess return of a portfolio that is long one nominal bond and short one TIPS bond with the same maturity.

Table 1 shows the results of regressing the 3-month overlapping excess returns on nominal bonds, TIPS, and breakeven inflation on the log growth of crude oil spot prices, the corresponding term spread, and the CPI less energy inflation. Columns (1) and (3) show that the oil price growth rate g_t^{Oil} is a significant explanatory variable for contemporaneous excess returns on nominal bonds and breakeven inflation. Increases in the oil price are associated with decreases in the expected excess return on nominal bonds and breakeven inflation, implying that realized breakeven inflation and nominal yields are higher. In addition, g_t^{Oil} contributes additional explanatory power over and beyond the term spread and the CPI less energy inflation, as reflected by an increase of 9% in the adjusted R^2.

Table 2 shows the results of predictive regressions. Column (3) shows that oil price change g_t^{Oil} is a significant predictor for the excess returns on breakeven inflation. However, oil price change fails to predict excess returns on Treasury bonds and TIPS. This finding of no predictability of oil price changes on nominal bond risk premium is counter intuitive, especially given the strong predictive relationship between oil price changes and equity risk premium (Driesprong, Jacobsen, and Maat, 2008).

In addition, g_t^{Oil} contributes additional forecasting power over and beyond the term spread and the CPI less energy inflation, reflected by an increase of 11% in the adjusted R^2. The forecasting power of g_t^{Oil} is also economically significant. For instance, a one-

[7] Because the TIPS market is less liquid than the Treasury market, especially in the early years of the TIPS market and during the 2007 financial crisis, TIPS bonds are priced lower (equivalently, TIPS yields are priced higher) to compensate for the liquidity risk. The liquidity-adjusted TIPS yield is estimated as $y_{n,t}^{TIPS,adj} = y_{n,t}^{TIPS} - L_{n,t}$, where $L_{n,t}$ is the liquidity premium, as in Pflueger and Viceira (2016).

standard-deviation increase in oil price (around 16% at quarterly frequency) predicts a 0.8% increase in the expected excess return on breakeven inflation. Given that the expected excess returns on breakeven inflation could be viewed as inflation risk premia, the above forecasting regression results indicate that the oil price change is a significant predictor for the inflation risk premia.

Interpreting the above empirical evidence, however, is challenging because the economic signal of oil price changes is ambiguous. Kilian (2009) shows that the impact of oil price shocks on the economy depends on the type of fundamental shocks that drive oil prices.

2.4 Empirical evidence on bond returns using oil supply, global demand, and oil-specific demand shocks

I proceed the examination of the impact of the three shocks on the excess returns of nominal bond, TIPS, and breakeven inflation.

Table 3 shows the results of regressing the 3-month overlapping excess returns of nominal bonds, TIPS, and breakeven inflation on the three shocks and other control variables. Oil supply shock is a significant explanatory variable for excess returns on Treasury bonds, TIPS, and breakeven inflation. On the other hand, global demand shock explains excess returns on Treasury bonds and breakeven inflation, but not on TIPS. Oil-specific demand shock has no explanatory power. Increases in oil prices, irrespective of oil supply or global demand shocks, are associated with negative excess returns on Treasury bonds, TIPS, and breakeven inflation, consistent with regression results using the oil price changes.

Table 4 shows the results of predictive regressions. Interestingly, none of the three shocks could predict excess returns on nominal bonds. Oil supply shocks weakly predict excess returns on breakeven inflation with a positive slope. Global demand shocks predict excess returns on TIPS with a negative slope and predict excess returns on breakeven inflation with a positive slope. In other words, global demand shocks predict negative real bond risk premium and positive inflation risk premium. Since these two effects offset each other, we observe insignificant effect on the bond risk premium. Oil-specific demand shocks show no predictability for either excess return.

These results highlight the importance of disentangling oil price changes into fundamental shocks in the oil market. Although oil price changes have no predictive power for excess returns on TIPS, global demand-driven oil price increases significantly predict negative excess returns on TIPS. The positive slopes for oil supply shocks and global demand shocks are consistent with the positive slope estimated in the predictive regression of excess returns on breakeven inflation on the oil price change.

In sum, the above empirical evidence shows that crude oil price changes contain valuable information for bond returns and breakeven inflation. One caveat exists: empirical tests on real bond returns and breakeven inflation are constrained by the short history of inflation-indexed bonds and inflation swap rates in the U.S. This caveat warrants theoretical studies to fully understand the impact of fundamental shocks in the oil market on bond returns and breakeven inflation. I proceed with a theoretical analysis in a two-sector New Keynesian model.

3 A two-sector New Keynesian model

The modeling framework builds on the workhorse New Keynesian model (Galí, 2008), which is the most suitable DSGE framework for analyzing nominal bond yields, real bond yields, and inflation processes; and their interactions with economic fluctuations.

There are three important departures from the standard New Keynesian model. First, an oil sector is included in addition to the standard consumption goods sector. The two sectors are labeled as the oil sector and the core sector. Oil and core goods are produced by a representative oil firm and monopolistic core goods firms, respectively. The inflation of oil prices represents energy inflation, while the inflation of core goods prices represents core inflation. Second, oil is included in the household utility function, to capture the fact that households spend about 4% of their pre-tax income on gasoline for transportation needs. In addition, household consumption of oil is assumed to be complementary to the consumption of core goods, as in Hitzemann (2016) and Ready (2017).[8] Third, oil is also used as an energy input in core goods firms' production functions, reflecting the fact that 40% of industrial energy comes from oil.

The oil price is assumed to be flexible, consistent with the average duration of 10 to 18 days between price changes in retail gasoline and of 2.4 days in wholesale gasoline (Douglas and Herrera, 2010). The core goods price is assumed to be sticky, supported by the fact of the average frequencies of 8 to 11 months of price changes of 350 product categories underlying the U.S. CPI (Nakamura and Steinsson, 2008). Last, the adjustment of real wages is assumed to be sluggish as in Blanchard and Galí (2007).

The productivity shock in the energy sector represents the oil supply shock. The productivity shock in the core sector is the supply shock in the core sector, but acts as demand shock in the oil market. Note that demand for oil comes from both households and core goods firms.

3.1 Households

An infinitely-lived representative household has recursive utility (Epstein and Zin, 1989; Weil, 1989):

$$V_t = (1-\beta)U(X_t, N_t)^{1-\rho} + \beta \left(E_t V_{t+1}^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}}, \tag{1}$$

[8] Oil is complementary to the consumption of some durable goods, such as motor vehicles. As the model does not distinguish between durable goods and non-durable goods, the complementarity of oil is modeled in a reduced form.

where β is the time discount factor, γ is the relative risk aversion, and $1/\rho$ is the elasticity of inter-temporal substitution (EIS). The period utility $U(X_t, N_t)$ is given by

$$U(X_t, N_t) = \left(\frac{X_t^{1-\rho}}{1-\rho} - \phi\kappa_t \frac{N_t^{1+\nu}}{1+\nu} \right)^{\frac{1}{1-\rho}}, \ \phi > 0, \ \nu > 0, \tag{2}$$

where X_t is the consumption bundle of oil and the final core goods, N_t is household labor supply to intermediate core goods firms, and $1/\nu$ is the Frisch elasticity of labor supply. The process κ_t is chosen to ensure balanced growth and will be specified in the core sector below. As the household values leisure, there is disutility from supplying labor to the intermediate goods firms.

The consumption bundle is a constant elasticity of substitution (CES) aggregation of oil and the final core goods:

$$X_t \equiv [(1-\xi)C_t^{1-\frac{1}{\eta}} + \xi(O_t^H)^{1-\frac{1}{\eta}}]^{\frac{1}{1-\frac{1}{\eta}}}, \tag{3}$$

where C_t is the final core goods, O_t^H is the oil directly consumed by the household, ξ measures the weight of O_t^H in the consumption bundle, and η measures the elasticity of substitution between oil and the final core goods.

The price of the consumption bundle is defined as

$$P_t^X \equiv [(1-\xi)(P_t^C)^{1-\eta} + \xi(P_t^O)^{1-\eta}]^{1/(1-\eta)}, \tag{4}$$

where P_t^C and P_t^O are the price of the final core goods and oil, respectively. It can be shown that $(1-\xi)C_t P_t^C + \xi O_t^H P_t^O = X_t P_t^X$.

The representative household is endowed with all the shares of the oil firm and core goods firms and receives dividends from the oil sector and the core sector. In addition, the household can trade one-period riskless bonds available in zero net supply. The household maximizes the utility given in equation (1) by choosing the optimal consumption of the final core goods and oil, the quantity of bonds, and labor supply:

$$\max_{C_t, O_t^H, B_t, N_t} V_t, \tag{5}$$

subject to the intertemporal budget constraint

$$X_t P_t^X + B_t \leq B_{t-1} R_{t-1} + W_t N_t + D_t^C + D_t^O, \tag{6}$$

where B_t is the quantity of bonds, W_t is the wage, and D_t^C and D_t^O are the dividends from the intermediate core sector and the oil sector, respectively.

The numéraire in the model is the one-period riskless bond. The bond costs one dollar in the period t and pays R_t dollars in the next period $t+1$. Thus, R_t corresponds to the gross nominal interest rate.

Following Blanchard and Galí (2007), I model real wage rigidities in a reduced way without specifying the exact friction in the labor market. The process of real wages is given by

$$\frac{W_t}{P_t^C} = \left(\frac{W_{t-1}}{P_{t-1}^C}\right)^{\rho_w} \left(-\frac{U_{N,t}}{U_{C,t}}\right)^{1-\rho_w}, \tag{7}$$

where ρ_w is an index of real wage rigidities and $-U_{N,t}/U_{C,t}$ is the marginal rate of intratemporal substitution between the labor supply and consumption of core goods. The higher the value of ρ_w, the more sluggish the adjustment of real wages.

The stochastic discount factor (SDF) is derived from the optimization of the household's problem. The one-period real SDF $M_{t,t+1}^R$ is the marginal rate of substitution between time t and time $t+1$

$$M_{t,t+1}^R = \beta \left(\frac{X_{t+1}}{X_t}\right)^{\frac{1}{\eta}-\rho} \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\eta}} \left(\frac{V_{t+1}^{1-\rho}}{\left(E_t V_{t+1}^{(1-\gamma)/(1-\rho)}\right)^{1/(1-\gamma)}}\right)^{\rho-\gamma}. \tag{8}$$

The one-period nominal SDF is defined as $M^{\$}_{t,t+1} \equiv M^{R}_{t,t+1} \frac{P^C_t}{P^C_{t+1}}$. One critical feature of the SDF is its dependence on the quantity of oil directly consumed by the household through the consumption bundle X_t.

3.2 Oil sector

A representative oil firm produces oil. As in Kogan, Livdan, and Yaron (2009), the production function of the oil firm takes a simple form:

$$Y^O_t = Z^O_t K^O_{t-1}, \tag{9}$$

where K^O_{t-1} is the capital stock, and Z^O_t is the total factor productivity (TFP) in the oil sector.

It is assumed that $z^o_t \equiv log Z^O_t$ follows an AR(1) process

$$z^o_t = \rho_o z^o_{t-1} + \sigma_o \varepsilon^o_t, \tag{10}$$

where $\varepsilon^o_t \sim i.i.d.N(0,1)$.

The law of motion for capital is given by

$$K^O_t = (1-\delta^o)K^O_{t-1} + \Phi^O\left(\frac{I^O_t}{K^O_{t-1}}\right)K^O_{t-1}, \tag{11}$$

$$\Phi^O\left(\frac{I^O_t}{K^O_{t-1}}\right) = \frac{b^o}{1-1/\zeta^o}\left(\frac{I^O_t}{K^O_{t-1}}\right)^{1-1/\zeta^o} + g^o, \tag{12}$$

where I^O_t is the new investment, δ^o is the depreciation rate of existing capital, and the function $\Phi^O(I^O_t/K^O_{t-1})$ is a positive, concave function, as in Jermann (1998). The parameter ζ^o is the elasticity of the investment capital ratio with respect to Tobin's q.

Oil is sold to the intermediate goods firms to produce intermediate goods and to the households for their consumption.[9] As the oil firm faces no price adjustment costs, the oil price P_t^O is flexible, consistent with the fact that the average duration between price changes in wholesale gasoline (retail gasoline) is 2.4 days (10 to 18 days) (Douglas and Herrera, 2010).

Given the oil price of P_t^O and the final core goods price of P_t^C, the oil firm chooses the optimal investment to maximize its firm value:

$$V_t^O \equiv \max_{I_t^O} E_t \sum_{j=0}^{\infty} M_{t,t+j}^{\$} D_{t+j}^O, \tag{13}$$

where $D_{t+j}^O \equiv Y_{t+j}^O P_{t+j}^O - I_{t+j}^O P_{t+j}^C$ is the dividend in period $t+j$ and $M_{t,t+j}^{\$}$ is the nominal SDF derived from the household's optimality conditions. The oil firm dividend goes to the household.

3.3 Core sector

The core sector is comprised of a final core goods firm and a continuum of monopolistic intermediate core goods firms.

3.3.1 Final core goods

A representative final core goods firm combines a continuum of intermediate core goods into the final core goods. The final core goods firm operates in a perfectly competitive market and thus is a price taker. The firm uses a constant elasticity of substitution (CES)

[9] For the sake of simplicity, the model abstracts from the oil inventory and the oil cartel. Carlson, Khokher, and Titman (2007) and Kogan, Livdan, and Yaron (2009) do not consider these two features in their models either. Inventory is not critical in the model, but the presence of inventory would mitigate the magnitude of oil supply and demand shocks on oil spot prices.

production technology to produce the final core goods:

$$Y_t^C \equiv \left(\int_0^1 (Y_t^C(i))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{14}$$

where $Y_t^C(i)$ is the quantity of intermediate core goods i, $i \in [0,1]$. The parameter ε measures the elasticity of substitution between intermediate core goods.

The final core goods are either consumed by the household or used as investment for new capital by the oil firm and intermediate core goods firms:

$$C_t + I_t^O + \int_0^1 I_t^C(i) di \leq Y_t^C, \tag{15}$$

where $I_t^C(i)$ is the investment made by the intermediate core goods firm i.

Given the final core goods price of P_t^C and the price of intermediate core goods i of $P_t^C(i)$, the final core goods firm maximizes its profit by choosing the optimal demand of core goods i:

$$\max_{Y_t^C(i)} P_t^C Y_t^C - \int_0^1 P_t^C(i) Y_t^C(i) di. \tag{16}$$

Furthermore, the optimal demand for the intermediate core goods i can be expressed as

$$Y_t^C(i) = \left(\frac{P_t^C(i)}{P_t^C} \right)^{-\varepsilon} Y_t^C. \tag{17}$$

Equations (14) and (17) together imply that the final core goods price is an aggregate price index of intermediate core goods prices, i.e., $P_t^C \equiv [\int_0^1 (P_t^C(i))^{1-\varepsilon} di]^{\frac{1}{1-\varepsilon}}$. Furthermore, it can be shown that $\int_0^1 P_t^C(i) Y_t^C(i) di = P_t^C Y_t^C$.

3.3.2 Intermediate core goods

Intermediate core goods are produced by a continuum of monopolistic firms indexed by $i \in [0,1]$. The production of intermediate core goods i is given by

$$Y_t^C(i) = [K_{t-1}^C(i)]^{\omega}[Z_t^C N_t(i)]^{\alpha}[O_t^I(i)]^{1-\alpha-\omega}, \tag{18}$$

where Z_t^C is the common productivity across all intermediate core goods firms, $K_{t-1}^C(i)$ is the capital stock, $N_t(i)$ is the labor employed, and $O_t^I(i)$ is the quantity of oil used in production. The oil share of production is measured by $1-\alpha-\omega$.

The law of motion for capital is given by

$$K_t^C(i) = (1-\delta^c)K_{t-1}^C(i) + \Phi^C\left(\frac{I_t^C(i)}{K_{t-1}^C(i)}\right)K_{t-1}^C(i), \tag{19}$$

$$\Phi^C\left(\frac{I_t^C(i)}{K_{t-1}^C(i)}\right) = \frac{b^c}{1-1/\zeta^c}\left(\frac{I_t^C(i)}{K_{t-1}^C(i)}\right)^{1-1/\zeta^c} + g^c, \tag{20}$$

where $I_t^C(i)$ is the new investment, δ^c is the depreciation rate of existing capital, and the function $\Phi^C(I_t^C(i)/K_{t-1}^C(i))$ is a positive, concave function, as in Jermann (1998). The parameter ζ^c represents the elasticity of the investment capital ratio with respect to Tobin's q.

Following Croce (2014), I assume that the productivity growth rate in the core sector, $\Delta z_{t+1}^c \equiv log(Z_{t+1}^C/Z_t^C)$, has both a long-run and a short-run risk component:

$$\Delta z_{t+1}^c = x_t^c + \sigma_c \varepsilon_{t+1}^c, \tag{21}$$

$$x_t^c = \rho_{xc} x_{t-1}^c + \sigma_{xc}\varepsilon_t^{xc}, \tag{22}$$

where $\varepsilon_t^c \sim i.i.d.N(0,1)$ and $\varepsilon_t^{xc} \sim i.i.d.N(0,1)$. In addition, all shocks are assumed to be mutually independent.

Following Rotemberg (1982) and Ireland (1997), I assume that each monopolistic firm changes its price every period but faces a real quadratic cost of price changes:

$$\Gamma(P_t^C(i)) \equiv \frac{\vartheta}{2}\left(\frac{P_t^C(i)}{\overline{\pi}P_{t-1}^C(i)} - 1\right)^2 Y_t^C. \tag{23}$$

The parameter ϑ measures the degree of price stickiness, which is common to all intermediate core goods firms. The variable $\overline{\pi}$ is the target gross inflation rate in the steady state. If the price grows at the rate of target inflation, the cost of the price adjustment is zero. If $\vartheta = 0$, there is no adjustment cost of price changes. Because of the quadratic cost of price changes, fewer final goods are available for consumption and investment. The presence of nominal price rigidity leads to inefficiency.[10]

As shown in (17), the optimal demand for the intermediate core goods i is downward-sloping, which is determined by the relative prices. Monopolistic firm i maximizes its firm value by choosing the optimal price of its goods and the optimal investment:

$$V_t^C(i) \equiv \max_{P_t^C(i), I_t^C(i)} \sum_{j=0}^{\infty} E_t M_{t,t+j}^{\$} D_{t+j}^C(i), \tag{24}$$

where $D_{t+j}^C(i) \equiv Y_{t+j}^C(i)P_{t+j}^C(i) - \Psi_{t+j}(Y_{t+j}^C(i)) - \Gamma(P_{t+j}^C(i))P_{t+j}^C - I_{t+j}^C(i)P_{t+j}^C$ is the dividend in period $t+j$ and $M_{t,t+j}^{\$}$ is the nominal SDF derived from the household's optimality conditions. The production cost function $\Psi_{t+j}(Y_{t+j}^C(i))$ for a given level output $Y_{t+j}^C(i)$ is defined below.

Given the oil price P_t^O and the wage W_t, the firm i minimizes production cost by

[10] An equivalent inflation dynamic can also be derived under the assumption of a staggered price-setting mechanism (Calvo, 1983). Ascari, Castelnuovo, and Rossi (2011) discusses the similarities and differences between the two approaches. The Rotemberg approach is better than the Calvo approach for replicating the dynamics of inflation at the macro level. An advantage of the assumption of quadratic price adjustment costs is that it leads to a tractable symmetric equilibrium. Because of the presence of nominal rigidity, real quantities depend on nominal prices and the nominal interest rate, which is governed by monetary policy.

choosing the optimal labor and oil:

$$\min_{N_t(i),O_t^I(i)} \Psi(Y_t^C(i)) \equiv W_t N_t(i) + O_t^I(i) P_t^O,$$
$$s.t.\ \ Y_t^C(i) = [K_{t-1}^C(i)]^{\omega}[Z_t^C N_t(i)]^{\alpha}[O_t^I(i)]^{1-\alpha-\omega}. \tag{25}$$

Last, the process κ_t is defined as $\kappa_t \equiv (Z_{t-1}^C)^{1-\rho}$ to ensure balanced growth.

3.4 Central bank

To complete the model, it is assumed that the central bank follows the Taylor rule in setting the nominal interest rate:

$$R_t = \bar{R}\left(\frac{\pi_t^{CPI}}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y_t^C}{\bar{Y}}\right)^{\phi_y},\ \phi_\pi \geq 0,\ \phi_y \geq 0, \tag{26}$$

where $\bar{R}$, $\bar{\pi}$, and $\bar{Y}$ represent the gross interest rate, the target gross total inflation, and the output of core goods in steady state, respectively.

3.5 Symmetric equilibrium

The equilibrium of the model is characterized by the solutions of the household's problem (5), the oil firm's problem (13), the final core goods firm's problem (16), and the intermediate core goods firms' problems (24). The first order conditions of these problems are presented in Appendix A.

The equilibrium turns out to be symmetric. Because all intermediate core goods firms have the identical cost minimization problem and the identical value maximization problem, they choose the same optimal demand for labor and oil, i.e., $N_t(i) = N_t$ and $O_t^I(i) = O_t^I$. Furthermore, they choose the same optimal selling price and investment, i.e., $P_t^C(i) = P_t^C$ and $I_t^C(i) = I_t^C$. In addition, all markets are clear.

3.6 Bond pricing and implied breakeven inflation

This sub-section first defines three measures of inflation and then uses the SDF to price zero-coupon nominal bonds, zero-coupon real bonds, and zero-coupon inflation swap contracts.

In the model, the core CPI price index, the energy CPI price index, and the CPI price index are represented by the prices of the final core goods, oil, and the consumption bundle, respectively. Let $\pi_t^C \equiv P_t^C / P_{t-1}^C$ denote core CPI inflation, $\pi_t^O \equiv P_t^O / P_{t-1}^O$ denote energy CPI inflation, and $\pi_t^{CPI} \equiv P_t^X / P_{t-1}^X$ denote CPI inflation.

The core inflation equation can be expressed in the log-linearization form

$$\widetilde{\pi}_t^c = \beta E_t \widetilde{\pi}_{t+1}^c + \lambda \widetilde{\psi}_t, \tag{27}$$

where $\lambda \equiv \frac{\varepsilon - 1}{\vartheta}$ is decreasing in the index of price stickiness ϑ, ψ_t represents the real marginal production cost of intermediate goods, and tilde variables denote the log deviation from steady state. Equation (27) shows that core inflation depends on the expected core inflation in the next period and the change of the real marginal production cost. The real marginal production cost of intermediate core goods is given by

$$\widetilde{\psi}_t = \frac{1 - \alpha - \omega}{1 - \omega} \widetilde{p}_t^o + \frac{\alpha}{1 - \omega} \widetilde{w}_t - \frac{1}{1 - \omega} \widetilde{z}_t^c + \frac{\omega}{1 - \omega} (\widetilde{y}_t^c - k_{t-1}^c). \tag{28}$$

Equations (27) and (28) show that the oil price change affects core inflation through the real marginal production cost of intermediate core goods.

Zero-coupon nominal bonds, zero-coupon real bonds, and zero-coupon inflation swaps are priced by the pricing kernel derived from the optimality conditions of the household's problem.

The yield of an n-year zero-coupon nominal Treasury bond is defined as the n-year nominal yield, given by

$$y_t^n = -\frac{1}{n}E_t(m_{t,t+n}^{\$}) - \frac{1}{2n}Var_t(m_{t,t+n}^{\$}), \tag{29}$$

where $m_{t,t+n}^{\$} \equiv logM_{t,t+n}^{\$}$.

The yield of an n-year zero-coupon real Treasury bond is defined as the n-year real yield, given by

$$r_t^n = -\frac{1}{n}E_t(m_{t,t+n}^{R,X}) - \frac{1}{2n}Var_t(m_{t,t+n}^{R,X}), \tag{30}$$

where $m_{t,t+n}^{R,X} \equiv logM_{t,t+n}^{R,X}$.

The n-year breakeven inflation is the difference between the n-year nominal yield and the n-year real yield, i.e., $y_t^n - r_t^n$. In addition, the n-year breakeven inflation rate also represents the n-year inflation swap rate.

Alternatively, the zero-coupon inflation swap rate can be directly estimated. When an inflation swap contract is initialized, the present value of expected cash flow at maturity should be zero. Assuming that the notional amount is one dollar and that the inflation index refers to the CPI index, the zero-coupon inflation swap fixed rate s_t^n is given by

$$0 = -E_t[M_{t,t+n}^{\$}e^{ns_t^n}] + E_t[M_{t,t+n}^{\$}\pi_{t,t+n}^{CPI}]. \tag{31}$$

Note that s_t^n is known at time t and the real SDF $M_{t,t+n}^{R,X} \equiv M_{t,t+n}^{\$}\pi_{t,t+n}^{CPI}$.

The n-year inflation swap rate could be further expressed as

$$s_t^n = \frac{1}{n}E_t\hat{\pi}_{t,t+n}^{CPI} - \frac{1}{2n}Var_t\hat{\pi}_{t,t+n}^{CPI} + \frac{1}{n}Cov_t(m_{t,t+n}^{R,X}, \hat{\pi}_{t,t+n}^{CPI}), \tag{32}$$

where $\hat{\pi}_{t,t+n}^{CPI} \equiv log\pi_{t,t+n}^{CPI}$ denotes the log CPI inflation. On the right-hand side of the equation, the first term is the expected inflation, the second term is the Jensen's inequality adjustment of the expected inflation, and the third term is the inflation risk premium. The inflation swap rate (ignoring the Jensen's inequality adjustment) consists of the inflation expectation and the inflation risk premium.

At this point, it is worth discussing scenarios that might lead to positive inflation risk premium. If inflation is high in "bad" states, where marginal utility is high, the covariance term will be positive. The inflation risk premium will be positive and the swap rate will be higher than the expected inflation if inflation is positively correlated with the real SDF. The fixed receiver in an inflation swap contract demands a higher rate to compensate for the risk of the realization of unexpected high inflation in "bad" states.

4 Quantitative results

The model is solved in Dynare using a second-order approximation at a quarterly frequency. The choices of parameter values are discussed first, followed by the implications for the oil market.

4.1 Calibration

Table 5 reports the parameter values used in the baseline calibration of the model. Parameter values are either set to those reported in previous studies whenever possible or chosen by matching the selected moments in the data. Parameters are grouped into four categories.

The time discount rate β takes a value of 0.997, which corresponds to an annual real interest rate of 1.2% in the long run. The relative risk aversion γ takes a value of 10. The elasticity of intertemporal substitution $1/\rho$ is set at 2, implying that households prefer an early resolution of uncertainty. The weight of oil in the consumption bundle ξ is set at 0.1, close to the weight of energy components in the CPI basket. The elasticity of substitution between oil and core goods η is set at 0.25, the same value as used in Ready (2017), implying complementarity between the two types of consumption. The real wage rigidity index ρ_w is set at 0.965 to match the ratio of $\sigma(\Delta w)/\sigma(\Delta y) = 0.44$, a key moment of wages in the data. The labor supply N is fixed at 0.33 in the deterministic steady state so that households spend one-third of their discretionary time working. The Frisch elasticity of labor supply is pinned down as 0.2498 by the the labor supply in the deterministic steady state.

Most parameters associated with production functions are set to values used in related papers. The constant elasticity of substitution of intermediate core goods ε is set at 6, which corresponds to a markup of 20%. The degree of capital adjustment cost (ζ^o and ζ^c) is set at 4.8, both in the oil sector and intermediate goods sector. Free parameters b^o and g^o are set as $b^o = (\delta^o)^{1/\zeta^o}$ and $g^o = \frac{1}{1-\zeta^o}\delta^o$ such that there is no adjustment cost for the oil sector in the deterministic steady state. Similarly, free parameters b^c and g^c are chosen such that there is no adjustment cost for the intermediate goods firms in the deterministic steady state. In particular, $b^c = (\delta^c)^{1/\zeta^c}$ and $g^c = \frac{1}{1-\zeta^c}\delta^c$. A higher value of 0.05 is set for the depreciation rate of the oil capital δ^o, which corresponds to an annualized rate of 20%. The depreciation rate of consumption goods capital δ^c is equal to 0.02. The capital share ω and labor share of output α are set at 0.33 and 0.57, respectively. The degree of price adjustment cost ϑ is set at 25, which is close to the values suggested by Ireland (2000).

Parameter values of the three productivity shocks are chosen to match the moments of the relative oil prices, core inflation, CPI inflation, 10-year nominal yields, household consumption, and wages, as listed in Table 6. The autocorrelation coefficient and the standard deviation of productivity shocks in the oil sector are set at 0.45 and 9.5%, respectively. The autocorrelation coefficient of the long-run productivity shocks, and the standard deviations of short-run and long-run productivity shocks in the core sector are set at 0.85, 1.2%, and 0.07%, respectively.

Coefficients in the Taylor rule ϕ_π and ϕ_y are set at 1.5 and 0.125, respectively, which are standard values in the monetary literature. The target inflation $\bar{\pi}$ is set at 1.0092, which corresponds to an annual inflation rate of 3.68%, close to the historical average rate of U.S. CPI inflation.

Moments from data and the model are summarized in Table 6. The model is able to match most moments of the relative oil prices, core inflation, CPI inflation, 10-year nominal yields, and the correlation between oil prices and change changes and 10-year nominal yield changes. In addition, the model is also able to match key macroeconomic data, in addition to inflation data. The relative volatility of the consumption, wage, and output in the model are close to the empirical counterparts. In particular, the negative correlation between consumption growth and inflation from the model closely matches that negative correlation in the data.

4.2 Oil market implications

This sub-section discusses the responses of oil production, household consumption of core goods, household consumption of oil, the relative price of oil, core inflation, and CPI inflation to the three types of productivity shocks. Specifically, I consider a negative productivity shock (ε_t^o) in the oil sector, a positive short-run productivity shock (ε_t^c) and a positive long-run productivity shock (ε_t^{xc}) in the core sector, which represent three scenarios that tend to raise the relative price of oil. In the baseline calibration, the oil productivity process (z_t^o) in the oil sector is transitory ($\rho_o = 0.45$) and volatile ($\sigma_o = 9.5\%$ per quarter). The shock to the short-run productivity growth ε_t^c is less volatile ($\sigma_c = 1.2\%$ per quarter), and the process x^c is persistent ($\rho_{xc} = 0.85$) and even less volatile ($\sigma_{xc} = 0.07\%$ per quarter). The impulse response function of a variable to a particular shock plots the percentage deviation from the stochastic steady state. The size of the shock is one standard deviation of each productivity shock.

Figure 6 illustrates the impulse response functions of the set of variables to a negative productivity shock (ε_t^o) in the oil sector. For a 9.5% decrease in oil productivity, the oil production immediately decreases by 9.5%, and the relative price of oil jumps by 12.2%. Core inflation also increases by 0.44% because the rise of oil price leads to an increase in the production cost of core goods. The increase in core goods price is relatively small in part because the core goods price is sticky, and in part because the share of oil in production is the smallest among all the factors of production. As a result of the large increase in oil prices, CPI inflation increases by 0.58%, which is higher than the increase in core inflation of 0.44%. The state of economy after a big oil disruption is considered to be "bad" by households because less oil is available in the economy. Households reduce oil consumption, with a sharp drop of 4.5%, and consume fewer final core goods, resulting in a 1.4% drop. As the oil supply disruption is assumed to be transitory, oil production gradually recovers. The impact of oil supply on the economy disappears after four quarters.

Figure 7 shows the impulse response functions of the set of variables to a positive short-run productivity shock (ε_t^c) in the core sector. For a 1.2% increase in the short-run productivity, the productivity in the core sector jumps to a higher level and stays there afterward. The intermediate core goods firms produce more core goods but sell them at a lower price because the marginal production cost decreases. Core inflation initially decreases by 0.08% and recovers to the long-run value. Households consume more core goods by 0.6% and consume slightly more oil because the household consumption of oil is complementary to that of core goods. In the oil sector, the supply of oil is inelastic in the short run so the price of oil rises. The relative oil price increases by 1.3%. Overall, CPI inflation decreases. The economy after a positive short-run productivity shock in the core sector is in a "good" state for households because households consume more oil and core goods. After a positive short-run productivity shock in the core sector, oil price increases are associated with decreases in core inflation and CPI inflation.

Figure 8 shows the impulse response functions of the set of key variables to a positive long-run productivity shock (ε_t^{xc}) in the core sector. Given that the long-run component of core-sector productivity growth is persistent, a positive one-time shock of 0.07% to the growth rate in the current period continuously raises the level of productivity in the next period and thereafter. From the current period, intermediate core goods firms and the oil producer start to increase investment to fully take advantage of the long-lasting growth in core-sector productivity. The output of both core goods and oil steadily increases. Because more output is used for new capital in the current period, households initially reduce the consumption of core goods and oil by 0.04% and 0.06%, respectively, and then gradually increase the consumption of both goods afterward. Due to the strong demand for oil from intermediate core goods firms, the relative price of oil initially rises by 0.06% and continues to rise for about nine quarters before returning to the long-run level. Core inflation and CPI inflation increase by 0.01% initially and then keep declining for about 18 quarters before returning to the long-run value. Given that households increasingly consume more oil and core goods except for the initial period, the state of the economy after a positive long-run productivity shock in the core sector is considered "good" by households.

In the competitive oil market, the oil price quickly responds to either type of the three shocks. Because of the nominal price rigidity of core goods prices and the real wage rigidity, however, core inflation responds slowly to the three types of shocks. In addition, the magnitude of the responses of the core inflation and CPI inflation is also affected by the parameter values of the oil share in production and the elasticity of substitution between household consumption of oil and core goods. In the model, the three productivity shocks together can generate many distinct dynamics of oil prices, inflation, and household consumption, the last of which determines the stochastic discount factor and consequently affects bond prices.

5 Impact of oil supply and demand shocks on breakeven inflation, real yields, and nominal yields

This section discusses how real yields, breakeven inflation, and nominal yields respond differently to each productivity shock. It turns out that the impacts of the three productivity shocks on breakeven inflation, real yields, and nominal yields have different signs, sizes, and impact duration. In addition, using simulated data from the baseline model, it shows that the model is able to replicate the empirical results. Last, alternative calibrations illustrate the importance of three productivity shocks in explaining oil price changes and bond returns.

5.1 Impulse response functions of breakeven inflation, real yields, and nominal yields

Figure 9 shows that the 10-year real yield responds positively to a negative productivity shock (ε_t^o) in the oil sector, a positive short-run productivity shock (ε_t^c) in the core sector, and a positive long-run productivity shock (ε_t^{xc}) in the core sector. The positive response of the real yield to a positive short-run or long-run productivity shock in the core sector is straightforward. An increase in productivity in the core sector leads to positive growth of overall consumption afterward, which implies an increase in real yields. On the other hand, the positive response of the real yield to a negative productivity shock in the oil sector is counterintuitive. A decrease in the transitory oil productivity brings down current consumption, but households expect positive growth of overall consumption because the productivity in the oil sector quickly recovers. In addition, the effect of positive ε_t^c and ε_t^{xc} shocks on the real yields lasts over 20 quarters.

As shown in Figure 10, the 10-year breakeven inflation responds positively to a negative productivity shock (ε_t^o) in the oil sector, but responds negatively to a positive short-run productivity shock (ε_t^c) or to a positive long-run productivity shock (ε_t^{xc}) in the core sector. A negative productivity shock in the oil sector raises breakeven inflation because of the rise of CPI inflation after the shock, as discussed in previous section. The impact on breakeven inflation gradually diminishes. On the other hand, positive short-run or long-run productivity shocks in the core sector lower breakeven inflation because CPI inflation decreases along with decreasing marginal production cost of core goods, and both shocks have long-lasting impacts.

Note that the nominal yield is the sum of the real yield and breakeven inflation for any given maturity. Figure 11 plots impulse response functions of the 10-year nominal yields to the three productivity shocks. Because both real yields and breakeven inflation respond positively to the negative productivity shock in the oil sector, the nominal yields unambiguously increase. However, the response of nominal yields to productivity shocks in the core sector depends on the relative magnitude of the positive responses of real yields and the negative responses of breakeven inflation. Under the calibration of the baseline model, nominal yields decrease, especially in response to the positive short-run productivity shock in the core sector. Therefore, it can be seen that breakeven inflation plays an important role in the joint dynamics of oil prices and bond yields.

The conventional wisdom that increases in oil prices raise breakeven inflation and nominal yields is true only for the case in which the increase in oil price results from the disruption to the oil supply. On the contrary, demand-driven oil price increases are associated with decreases in breakeven inflation and nominal yields. More importantly, the model illustrates the necessity of identifying the type of shocks that cause oil price fluctuation, and the importance of decomposing nominal yields into real yields and breakeven inflation. The analysis on the interaction among distinct oil supply and demand shocks, nominal and real bond yields, and breakeven inflation is clear, informative, and comprehensive.

5.2 Model simulation

Using simulated data from the baseline model, Table 7 shows the contemporaneous regressions of excess returns on 10-year nominal bonds, real bonds, and breakeven inflation on oil price changes. The model is able to replicate the empirical results presented in Table 1; slope coefficients have the same signs and statistical significance levels while the R^2 are fairly large.

The coefficients on g_t^{Oil} in columns (1) and (3) in the data (Table 1) and in the model (Table 7) are negative and significant. In the model, both breakeven inflation and nominal yields increase after negative productivity shocks in the oil sector but decrease after positive productivity shocks in the core sector. Increases (decreases) in nominal yields and breakeven inflation lead to lower (higher) excess returns on nominal bonds and breakeven inflation, respectively. Negative significant slope coefficients on g_t^{Oil} in the regressions for nominal bonds and breakeven inflation suggest that the impact of productivity shocks in the oil sector dominates that of productivity shocks in the core sector, and otherwise the coefficient on oil price changes for the regression on breakeven inflation is expected to be positive.

When the price of oil rises, in the model, real yields increase for all shocks. Excess returns on real bonds thus respond negatively to positive oil price growth, so the slope coefficient on oil price growth should be negative. In column (2), the coefficient on g_t^{Oil} is negative in the data and simulated data. However, the negative slope coefficient in the model is significant, while it is insignificant in the data.

Admittedly, the current model is incapable of replicating the empirical predictive regressions. The reason for the lack of predictability is that the three productivity shocks are homoskedastic in the model, implying constant risk premia. Time-varying volatility of productivity shocks will be considered in future research.

5.3 Alternative calibrations and implications

To gain insight into the importance of productivity shocks in both sectors, I estimate model-implied statistics for alternative specifications. Table 8 reports three cases. Column (3) refers to a specification that the productivity shock in the oil sector is set to zero in the baseline model, i.e., $\sigma_o = 0$. Column (4) refers to a specification that the transitory productivity shock in the core sector is set to zero in the baseline model, i.e., $\sigma_c = 0$. Column (5) refers to a specification that the permanent productivity shock in the core sector is set to zero in the baseline model, i.e., $\sigma_{xc} = 0$. Column (3) indicates that the absence of oil supply shocks leads to a negative correlation between changes in 10-year yields and oil prices, which is at odds with data. Columns (4) and (5) indicate that the absence of oil demand shocks leads to mismatching inflation measures. To sum up, the three types of shocks in the two sectors are necessary and important elements of the model.

Table 9 reports unconditional variance decompositions for the baseline model. Consistent with the analysis above, the productivity shock in the oil sector accounts for the majority of variation in relative oil prices, CPI inflation, and core inflation. On the other hand, the long-run productivity shock in the core sector is important for long-term nominal and real yields and breakeven inflation. Lastly, the short-run productivity shock in the core sector also plays an important role in inflation, especially the breakeven inflation.

6 Oil and inflation relationships in the 2000s energy crisis and the Russian oil supply shock of 2022

The predictions of the two-sector New Keynesian oil model can help to explain (i) the muted impact of 2000s oil crisis on expected inflation and (ii) the comovement of expected inflation and the oil price surge that resulted from Russia's invasion of Ukraine in 2022.

As shown in Figure 1, both yields on U.S. 10-year Treasury securities and Treasury Inflation-Protected Securities (TIPS) remained stable at the low level when the crude oil prices experienced a persistent run-up from about US$30 per barrel in 2003 to about US$60 per barrel in 2005 and then to US$147 per barrel in 2008. Figure 12 shows the 10-year breakeven inflation exhibits stable movement in the period of 2003 to 2008. For the 2000s energy crisis, the strong demand for oil from emerging economies is considered as one of the major drivers of the long-run oil price hikes, while the slow growth in oil production is viewed as a contributing factor too. According to the model, for the increases in oil prices driven by the strong demand in the economy, there is downward pressure on the breakeven inflation; for the increases in oil prices driven by the slow growth in oil output, there is upward pressure on the breakeven inflation. These two effects may explain why the breakeven inflation and nominal bond yields remained stable in 2000s.

However, for the oil price surge that resulted from Russia's invasion of Ukraine since February 2022, the sanctions to Russian energy sector is the main cause, which is viewed as a negative shock to the world oil supply. As the model predicts, breakeven inflation rises in response to the negative oil supply shock. We thus observe the spike in breakeven inflation and nominal and real bond yields along with the surge of oil prices, as shown in Figure 13.

7 Conclusion

Rising oil prices directly affects the Consumer Price Index (CPI) because gasoline accounts for about 4-6% of the overall CPI and indirectly passes through to the prices of most goods and services because oil is required to produce or deliver these goods and services. Nevertheless, existing research shows that oil price changes have little or no predictive ability for short-term CPI inflation. Using oil supply, global demand, and oil-specific demand shocks, estimated from a structural VAR model of oil price changes, this paper first provides new empirical evidence that the dissecting of oil price changes improves CPI and core CPI inflation forecasts.

Compared with stocks, bonds are more directly affected by fluctuations in oil prices through the expected inflation component in nominal bond yields. Surprisingly, prior literature finds little predictive power of oil price changes on bond excess returns. This finding is counter intuitive, especially given the strong predictive relationship between oil price changes and equity risk premium. This paper further provides new empirical evidence that the oil price changes driven by global demand shocks predict negative real bond risk premium and positive inflation risk premium. Since these two effects offset each other, we observe insignificant effect on the bond risk premium.

This paper then builds a two-sector New Keynesian model to show theoretically that real yields, breakeven inflation, and nominal yields respond differently to oil supply and demand shocks. In the model, real bond yields respond positively to increases in oil prices, irrespective of the type of oil shocks; breakeven inflation responds positively to the rise of oil price driven by negative oil supply shocks, and responds negatively to the rise of oil price driven by positive productivity shocks in the economy. The model's predictions help to explain (i) the muted impact of 2000s oil crisis on expected inflation and (ii) the comovement of expected inflation and the oil price surge that resulted from Russia's invasion of Ukraine.

Does oil drive inflation? This is one of the key questions asked by the Fed and other central banks. The price of crude oil increased dramatically due to the war in Ukraine, and we observe the 41-year high inflation of 9.1% in June 2022. Although both oil prices and inflation have declined since their recent peaks since 2022, this paper highlights the necessity of identifying the type of oil price shocks and their impact on inflation forecasts and bond yields.

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Table 1. **Contemporaneous regressions of excess bond returns, excess TIPS returns, and breakeven inflation returns on oil price changes.** The log growth of crude oil spot prices is used to explain contemporaneous 3-month overlapping excess returns on U.S. 10-year Treasury nominal bonds, U.S. 10-year inflation-indexed bonds (TIPS), and breakeven inflation, in addition to the corresponding term spreads and the inflation of the CPI-All Items less Energy price index. Excess returns are defined in the text. The nominal term spread ($tms_t^{\$}$), liquidity adjusted TIPS term spread (tms_t^{TIPS}), and breakeven term spread (tms_t^{BE}) are the difference between 10-year and one-quarter nominal yields, the difference between 10-year and one-quarter TIPS yields, and the difference between 10-year and one-quarter breakeven inflation, respectively. U.S. 10-year TIPS yields and breakeven inflation are liquidity-adjusted as in Pflueger and Viceira (2016). g_t^{Oil} denotes the 3-month overlapping quarterly log growth of crude oil spot prices. π_t^{CPILE} is the quarterly inflation of the seasonally-adjusted Consumer Price Index - All Items less Energy. The sample period is 1999.6 - 2014.12. Standard errors are Newey-West adjusted with six lags. Standard errors are reported in brackets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1) $xr_t^{\$}$	(2) xr_t^{TIPS}	(3) xr_t^{BE}
g_t^{Oil}	-0.08***	-0.03	-0.05***
	(0.02)	(0.03)	(0.01)
$tms_t^{\$}$	2.45*		
	(1.51)		
tms_t^{TIPS}		2.37*	
		(1.69)	
tms_t^{BE}			1.28
			(1.47)
π_t^{CPILE}	1.56	1.34	-0.09
	(3.07)	(2.93)	(1.39)
Const.	-0.00	-0.00	0.00
	(0.02)	(0.02)	(0.01)
Adj. R^2 (*excl.* g^{Oil})	1.6%	0.5%	1.4%
Adj. R^2 (*incl.* g^{Oil})	10.9%	2.5%	10.9%

Table 2. **Predictive regressions of excess bond returns, excess TIPS returns, and breakeven inflation returns on oil price changes.** The log growth of crude oil spot prices is used to forecast 3-month overlapping log excess returns on 10-year U.S. Treasury nominal bonds, the liquidity-adjusted log excess returns on 10-year U.S. inflation-indexed bonds (TIPS), and the liquidity-adjusted log excess breakeven inflation returns, in addition to the corresponding term spreads and the inflation of the CPI-All Items less Energy price index. Excess returns are defined in the text. The nominal term spread ($tms_t^{\$}$), liquidity adjusted TIPS term spread (tms_t^{TIPS}), and breakeven term spread (tms_t^{BE}) are the difference between 10-year and one-quarter nominal yields, the difference between 10-year and one-quarter TIPS yields, and the difference between 10-year and one-quarter breakeven inflation, respectively. U.S. 10-year nominal and liquidity-adjusted TIPS yields, liquidity-adjusted breakeven inflation, and the liquidity risk premium are from Pflueger and Viceira (2016). π_t^{CPILE} is the quarterly inflation of the seasonally-adjusted Consumer Price Index - All Items less Energy. The sample period is 1999.6 - 2014.12. Standard errors are Newey-West adjusted with six lags. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1) $xr_{t+1}^{\$}$	(2) xr_{t+1}^{TIPS}	(3) xr_{t+1}^{BE}
g_t^{Oil}	0.01	-0.03	0.05**
	(0.02)	(0.02)	(0.01)
$tms_t^{\$}$	4.68**		
	(1.52)		
tms_t^{TIPS}		4.09**	
		(1.57)	
tms_t^{BE}			5.89**
			(1.58)
π_t^{CPILE}	6.02	2.64	3.05*
	(3.34)	(2.78)	(1.27)
Const.	-0.04	-0.02	-0.02**
	(0.02)	(0.02)	(0.01)
Adj. R^2 (*excl.* g^{Oil})	10.0%	4.3%	10.7%
Adj. R^2 (*incl.* g^{Oil})	9.7%	6.8%	21.6%

Table 3. **Contemporaneous regressions of excess bond returns, excess TIPS returns, and breakeven inflation returns on oil supply shocks, global demand shocks, and oil-specific demand shock.** Oil supply shocks (g_t^S), global demand shocks (g_t^{GD}), and oil-specific demand shocks (g_t^{OSD}) are used to explain contemporaneous 3-month overlapping excess returns on 10-year U.S. Treasury nominal bonds, 10-year U.S. inflation-indexed bonds (TIPS), and breakeven inflation, in addition to the corresponding term spreads and the inflation of the CPI-All Items less Energy price index. The oil supply, global demand, and oil-specific demand shocks are estimated in a structural VAR using the log growth rates of U.S. refiner acquisition cost of imported crude oil. Excess returns are defined in the text. The nominal term spread ($tms_t^{\$}$), liquidity adjusted TIPS term spread (tms_t^{TIPS}), and breakeven term spread (tms_t^{BE}) are the difference between 10-year and one-quarter nominal yields, the difference between 10-year and one-quarter TIPS yields, and the difference between 10-year and one-quarter breakeven inflation, respectively. U.S. 10-year TIPS yields and breakeven inflation are liquidity-adjusted. g_t^{Oil} denotes the 3-month overlapping quarterly log growth of crude oil spot prices. π_t^{CPILE} is the quarterly inflation of the seasonally-adjusted Consumer Price Index - All Items less Energy. The sample period is 1999.6 - 2014.12. Standard errors are Newey-West adjusted with six lags. Standard errors are reported in brackets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1) $xr_t^{\$}$	(2) xr_t^{TIPS}	(3) xr_t^{BE}
g_t^S	-0.13***	-0.08***	-0.06***
	(0.03)	(0.03)	(0.02)
g_t^{GD}	-0.08*	-0.03	-0.06**
	(0.05)	(0.06)	(0.03)
g_t^{OSD}	0.04	0.07	-0.02
	(0.05)	(0.05)	(0.04)
$tms_t^{\$}$	2.27		
	(1.45)		
tms_t^{TIPS}		2.52*	
		(1.41)	
tms_t^{BE}			0.83
			(1.51)
π_t^{CPILE}	1.64	1.63	-0.14
	(3.01)	(2.85)	(1.43)
Const.	-0.00	-0.01	0.00
	(0.02)	(0.02)	(0.01)
R^2	16.1%	7.2%	14.8%

Table 4. **Predictive regressions of excess bond returns, excess TIPS returns, and breakeven inflation returns on oil supply shocks, global demand shocks, and oil-specific demand shock.** Oil supply shocks (g_t^S), global demand shocks (g_t^{GD}), and oil-specific demand shocks (g_t^{OSD}) are used to forecast 3-month overlapping excess returns on 10-year U.S. Treasury nominal bonds, 10-year U.S. inflation-indexed bonds (TIPS), and breakeven inflation, in addition to the corresponding term spreads and the inflation of the CPI-All Items less Energy price index. The oil supply, global demand, and oil-specific demand shocks are estimated in a structural VAR using the log growth rates of U.S. refiner acquisition cost of imported crude oil. Excess returns are defined in the text. The nominal term spread ($tms_t^{\$}$), liquidity adjusted TIPS term spread (tms_t^{TIPS}), and breakeven term spread (tms_t^{BE}) are the difference between 10-year and one-quarter nominal yields, the difference between 10-year and one-quarter TIPS yields, and the difference between 10-year and one-quarter breakeven inflation, respectively. U.S. 10-year TIPS yields and breakeven inflation are liquidity-adjusted. π_t^{CPILE} is the quarterly inflation of the seasonally-adjusted Consumer Price Index - All Items less Energy. The sample period is 1999.6 - 2014.12. Standard errors are Newey-West adjusted with six lags. Standard errors are reported in brackets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1) $xr_{t+1}^{\$}$	(2) xr_{t+1}^{TIPS}	(3) xr_{t+1}^{BE}
g_t^S	0.04	-0.01	0.05*
	(0.05)	(0.03)	(0.03)
g_t^{GD}	-0.06	-0.11***	0.05**
	(0.04)	(0.04)	(0.03)
g_t^{OSD}	0.00	-0.03	0.03
	(0.07)	(0.05)	(0.04)
$tms_t^{\$}$	4.59***		
	(1.52)		
tms_t^{TIPS}		4.27***	
		(1.55)	
tms_t^{BE}			6.01***
			(1.88)
π_t^{CPILE}	5.77*	2.59	3.05**
	(3.38)	(2.86)	(1.45)
Const.	-0.04*	-0.02	-0.02***
	(0.02)	(0.02)	(0.01)
R^2	12.8%	11.4%	22.8%

Table 5. **Parameter values.** Parameter values are at a quarterly frequency. Parameters are grouped into four categories: preferences, production, shocks, and monetary policy.

Group	Description	Symbol	Value
Preferences	Time discount rate	β	0.997
	Relative risk aversion	γ	10
	EIS	$1/\rho$	2
	Coefficient of disutility	ϕ	3.6272
	Frisch elasticity of labor supply	ν	0.2498
	Oil share of the consumption bundle	ξ	0.1
	Elasticity of substitution of oil and core goods	η	0.25
	Index of real wage rigidity	ρ_w	0.965
	Labor supply in the DSS	N	0.33
Production	CES of intermediate core goods	ε	6
	Degree of price adjustment cost	ϑ	25
	Degree of oil capital adjustment cost	ζ^o	4.8
	Degree of core goods capital adjustment cost	ζ^c	4.8
	Depreciation rate of oil capital	δ^o	0.05
	Depreciation rate of core goods capital	δ^c	0.02
	Capital share of output	ω	0.33
	Labor share of output	α	0.568
	Oil share of output	$1-\alpha-\omega$	0.102
Shocks	z^o-shock in the DSS	$\bar{z^o}$	0
	AR(1) coefficient of z^o-shock	ρ_o	0.45
	Standard deviation of z^o-shock	σ_o	9.5%
	z^c-shock in the DSS	$\bar{z^c}$	0
	Standard deviation of SRR shock	σ_c	1.2%
	AR(1) coefficient of LRR shock	ρ_{xc}	0.85
	Standard deviation of LRR shock	σ_{xc}	0.07%
Policy	CPI inflation target	$\bar{\pi}$	1.0092
	Sensitivity of the interest rate to inflation	ϕ_π	1.5
	Sensitivity of the interest rate to output	ϕ_y	0.125

Table 6. **Moments.** This table reports the means, standard deviations, autocorrelations of growth rates of relative oil prices, core inflation, CPI inflation, 10-year nominal yields, correlations, and macroeconomic moments from the data and the model. The reported statistics from the data are numbers at a quarterly frequency for the period of 1987Q4 to 2014Q4. y^{10Y} refers to the 10-year nominal yields. g_t^{Oil} represents the growth rate of nominal oil prices. The model is calibrated at a quarterly frequency.

	Data	Model
Relative oil prices		
$E(\Delta log(P_t^O/P_t^C))$	0.45%	0.03%
$\sigma(\Delta log(P_t^O/P_t^C))$	18.48%	14.39%
$AC1(\Delta log(P_t^O/P_t^C))$	0.01	-0.22
Inflation		
$E(\pi_t^C)$	0.64%	0.64%
$\sigma(\pi_t^C)$	0.29%	0.58%
$AC1(\pi_t^C)$	0.72	0.57
$E(\pi_t^{CPI})$	0.66%	0.64%
$\sigma(\pi_t^{CPI})$	0.62%	0.67%
$AC1(\pi_t^{CPI})$	0.05	0.32
10Y nominal yields		
$E(y^{10Y})$	1.31%	0.94%
$\sigma(y^{10Y})$	1.01%	0.25%
$AC1(y^{10Y})$	0.94	0.89
Correlations		
$Corr(\Delta y_t^{10Y}, g_t^{Oil})$	0.40	0.49
$Corr(\Delta c, \pi^{CPI})$	-0.56	-0.50
Macroeconomic moments		
$\sigma(\Delta c)/\sigma(\Delta y^c)$	0.51	0.76
$\sigma(\Delta w)/\sigma(\Delta y^c)$	0.44	0.42

Table 7. **Contemporaneous regressions of excess bond returns using simulated data.** This table uses simulated data from the baseline model to replicate contemporaneous regressions as in Table 1. Each simulation generates a series of quarterly growth rates of oil prices, nominal yields, real yields, and breakeven inflation for 64 quarters. Regressions on these simulated data are repeated 3,000 times. The nominal term spread ($tms_t^{\$}$), TIPS term spread ($tms_t^{TIPS}$), and breakeven term spread (tms_t^{BE}) are the difference between 10-year and one-quarter nominal yields, the difference between 10-year and one-quarter TIPS yields, and the difference between 10-year and one-quarter breakeven inflation, respectively. $\pi_t^{CPI\ Core}$ is the quarterly inflation of Core Consumer Price Index. Standard errors are reported in brackets. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

	(1) $xr_t^{\$}$	(2) xr_t^{TIPS}	(3) xr_t^{BE}
g_t^{Oil}	-0.10***	-0.02***	-0.05**
	(0.03)	(0.01)	(0.02)
$tms_t^{\$}$	-4.35**		
	(2.01)		
tms_t^{TIPS}		1.53***	
		(0.41)	
tms_t^{BE}			-21.8***
			(5.58)
$\pi_t^{CPI\ Core}$	-5.39**	0.37	-8.10**
	(2.39)	(0.31)	(2.10)
Const.	0.04*	-0.00	0.08***
	(0.02)	(0.00)	(0.02)
R^2	47.4%	86.0%	42.6%

Table 8. **Data and model implied statistics for alternative specifications.** This table reports summary statistics for key variables from the data and models with alternative specifications. Column (2) is the baseline model. Column (3) refers to a specification that the productivity shock in the oil sector is set to zero in the baseline model, i.e., $\sigma_o = 0$. Column (4) refers to a specification that the transitory productivity shock in the core sector is set to zero in the baseline model, i.e., $\sigma_c = 0$. Last, column (5) refers to a specification that the permanent productivity shock in the core sector is set to zero in the baseline model, i.e., $\sigma_{xc} = 0$.

	(1) Data	(2) Model	(3) $\sigma_o = 0$	(4) $\sigma_c = 0$	(5) $\sigma_{xc} = 0$
Relative oil prices					
$E(\Delta log(P_t^O/P_t^C))$	0.45%	0.03%	0.01%	-0.05%	0.00%
$\sigma(\Delta log(P_t^O/P_t^C))$	18.48%	14.39%	1.35%	14.41%	14.55%
$AC1(\Delta log(P_t^O/P_t^C))$	0.01	-0.22	0.25	-0.24	-0.26
Inflation					
$E(\pi_t^C)$	0.64%	0.64%	0.65%	0.84%	0.58%
$\sigma(\pi_t^C)$	0.29%	0.58%	0.26%	0.54%	0.57%
$AC1(\pi_t^C)$	0.72	0.57	0.85	0.51	0.54
$E(\pi_t^{CPI})$	0.66%	0.64%	0.65%	0.84%	0.58%
$\sigma(\pi_t^{CPI})$	0.62%	0.67%	0.26%	0.64%	0.66%
$AC1(\pi_t^{CPI})$	0.05	0.32	0.89	0.29	0.33
10Y nominal yields					
$E(y^{10Y})$	1.31%	0.94%	0.98%	1.15%	0.95%
$\sigma(y^{10Y})$	1.01%	0.25%	0.18%	0.08%	0.16%
$AC1(y^{10Y})$	0.94	0.89	0.91	0.78	0.87
Correlations					
$Corr(\Delta y_t^{10y}, g_t^{Oil})$	0.40	0.49	-0.92	0.92	0.52
$Corr(\Delta c, \pi^{CPI})$	-0.56	-0.50	-0.10	-0.61	-0.51
Macroeconomic moments					
$\sigma(\Delta c)/\sigma(\Delta y^c)$	0.51	0.76	0.73	0.77	0.76
$\sigma(\Delta w)/\sigma(\Delta y^c)$	0.44	0.42	0.74	0.05	0.42

Table 9. **Variance decompositions for the baseline model.** This table reports the unconditional variance decompositions of the relative oil price, inflation, 10-year nominal yield, 10-year real yield, and 10-year breakeven inflation for the three productivity shocks ε^o, ε^c, and ε^{xc} in the baseline model. Variance decompositions are in percentage terms. The parameters values of the baseline model are given in Table 5.

	ε^o	ε^c	ε^{xc}
Relative oil price			
$log(P_t^O/P_t^C)$	92.05	7.33	0.62
Inflation			
π_t^{CPI}	70.23	25.46	4.30
π_t^C	63.05	31.65	5.29
10Y nominal yield			
y^{10Y}	1.90	81.41	16.70
10Y real yield			
r^{10Y}	43.55	30.47	25.98
10Y breakeven inflation			
s^{10Y}	0.43	81.39	18.18

Figure 1. **Crude Oil Prices, Yield on U.S. Treasury Securities at 10-Year Constant Maturity, and Yiled on U.S. Treasury Securities at 10-Year Constant Maturity, Inflation-Indexed.** This figure plots the crude oil prices (WTI-Cushing, Oklahoma) on the left y-axis, and yields on U.S. 10-Year Treasury and 10-Year TIPS on the right y-axis.

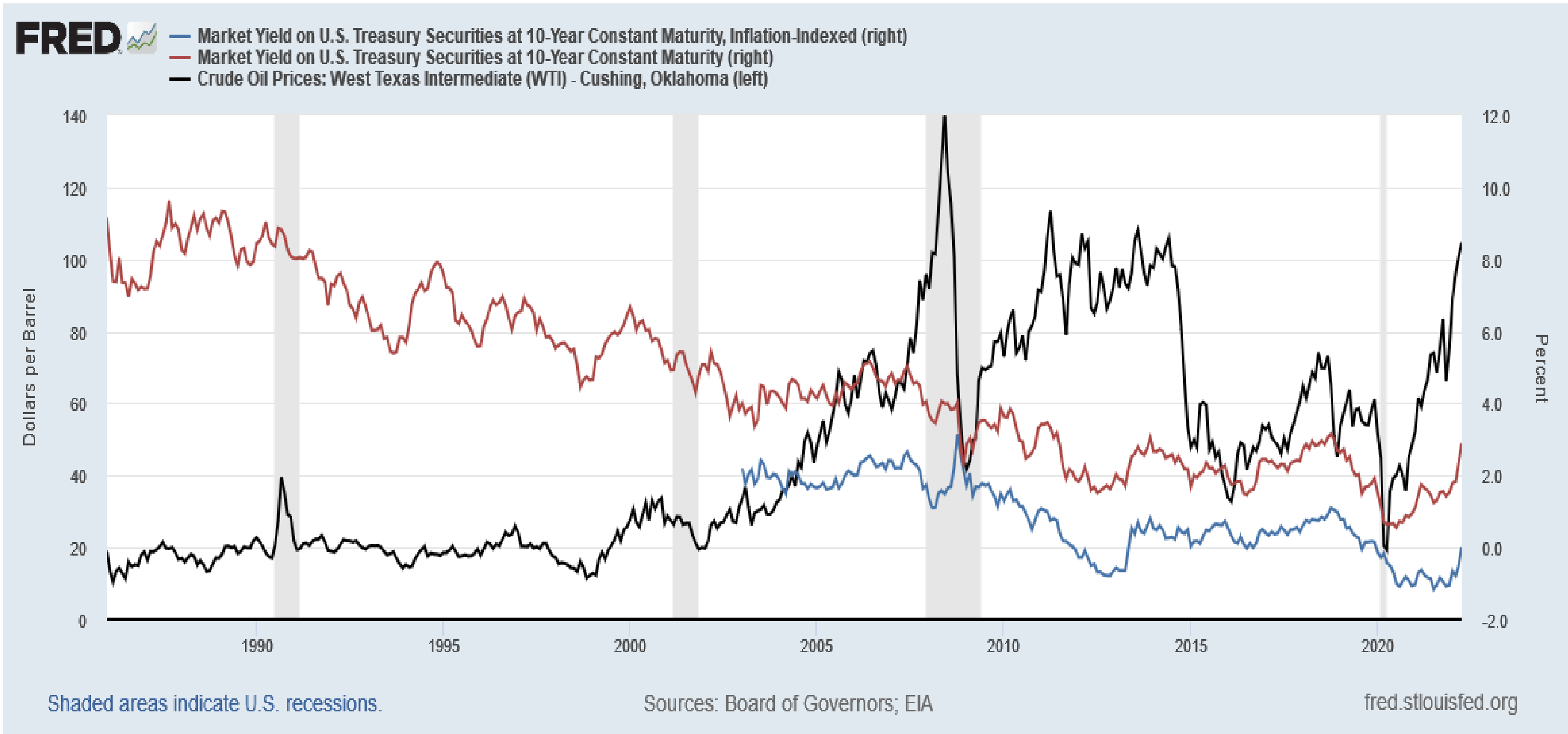


Figure 2. **Predictive Ability of Oil for CPI.** This figure shows the Root Mean Square Errors (RMSEs) of the forecasts of CPI inflation without oil prices, with oil prices, and with oil price shocks estimated in a structural VAR. The one-year ahead annual CPI inflation is predicted using the current and four lagged quarterly CPI inflation (annualized), or with the current and four lagged oil price growth rates, or with the current and four lagged oil supply shocks, global demand shocks, and oil-specific demand shocks.

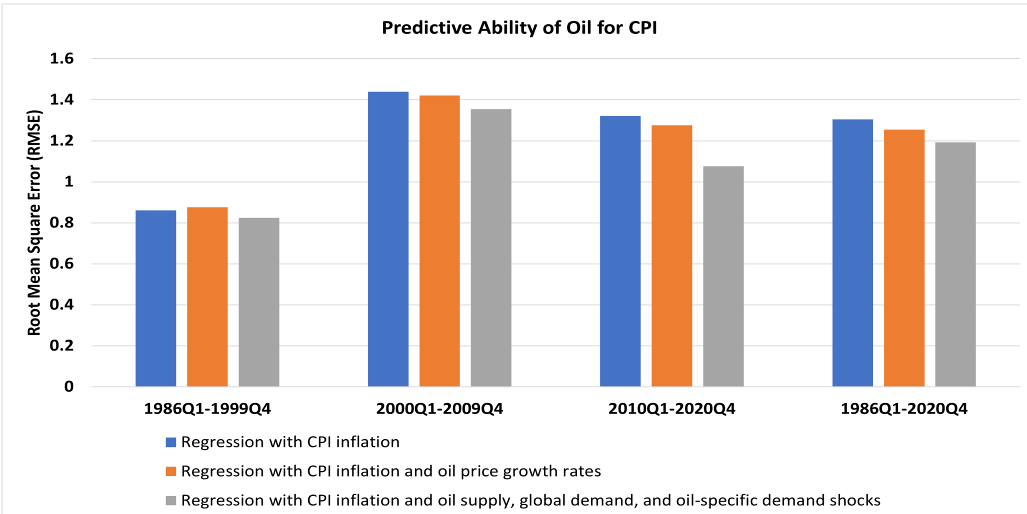


Figure 3. **Predictive Ability of Oil for Core CPI.** This figure shows the Root Mean Square Errors (RMSEs) of the forecasts of Core CPI inflation without oil prices, with oil prices, and with oil price shocks estimated in a structural VAR. The one-year ahead annual Core CPI inflation is predicted using the current and four lagged quarterly Core CPI inflation (annualized), or with the current and four lagged oil price growth rates, or with the current and four lagged oil supply shocks, global demand shocks, and oil-specific demand shocks.

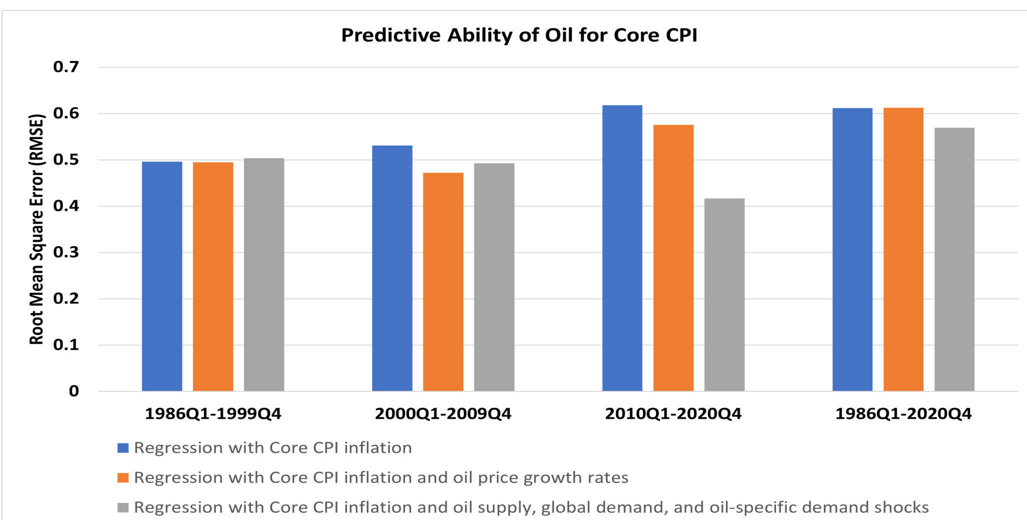


Figure 4. **Predictive Ability of Oil for CPI using UM Inflation Expectations.** This figure shows the Root Mean Square Errors (RMSEs) of the forecasts of CPI inflation without oil prices, with oil prices, and with oil price shocks estimated in a structural VAR. The one-year ahead annual CPI inflation is predicted using the median expectation for one-year-ahead inflation expectations from the University of Michigan's Survey of Consumers, or with the current and four lagged oil price growth rates, or with the current and four lagged oil supply shocks, global demand shocks, and oil-specific demand shocks.

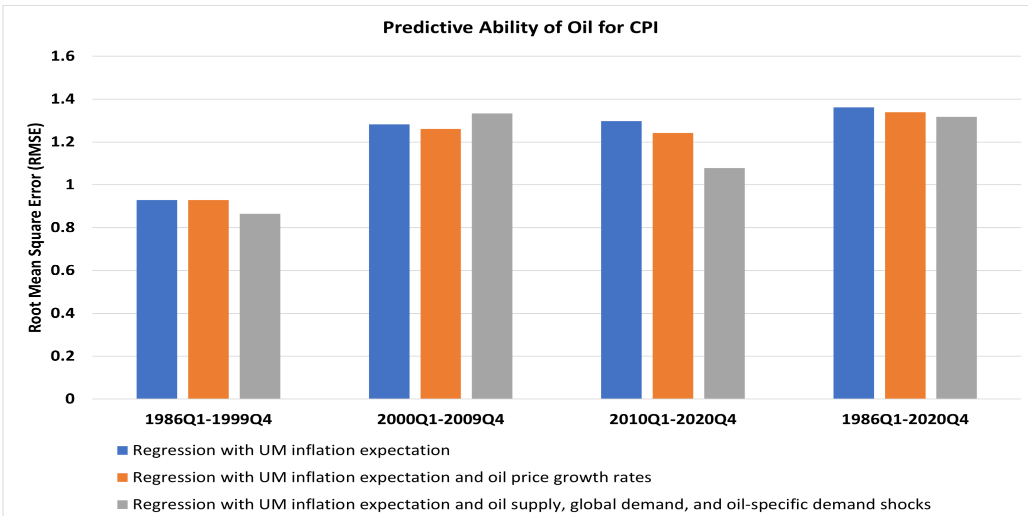


Figure 5. **Predictive Ability of Oil for CPI using SPF Inflation Expectations.** This figure shows the Root Mean Square Errors (RMSEs) of the forecasts of CPI inflation without oil prices, with oil prices, and with oil price shocks estimated in a structural VAR. The one-year ahead annual CPI inflation is predicted using the median expectation for the one-year-ahead CPI inflation from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF), or with the current and four lagged oil price growth rates, or with the current and four lagged oil supply shocks, global demand shocks, and oil-specific demand shocks.

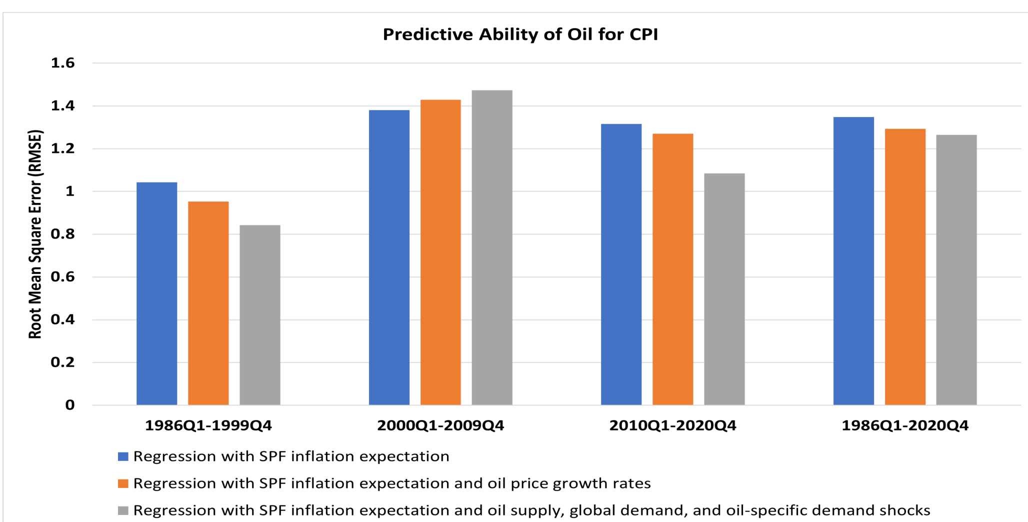


Figure 6. **Impulse response functions to a negative productivity shock (ε_t^o) in the oil sector.** This figure plots impulse response functions of oil production, household consumption of core goods, household consumption of oil, relative oil prices (P_t^O/P_t^C), core inflation, and CPI inflation. The size of the shock (ε_t^o) is one standard deviation $\sigma_o = 9.5\%$. The y-axis represents the percentage deviation from the steady state.

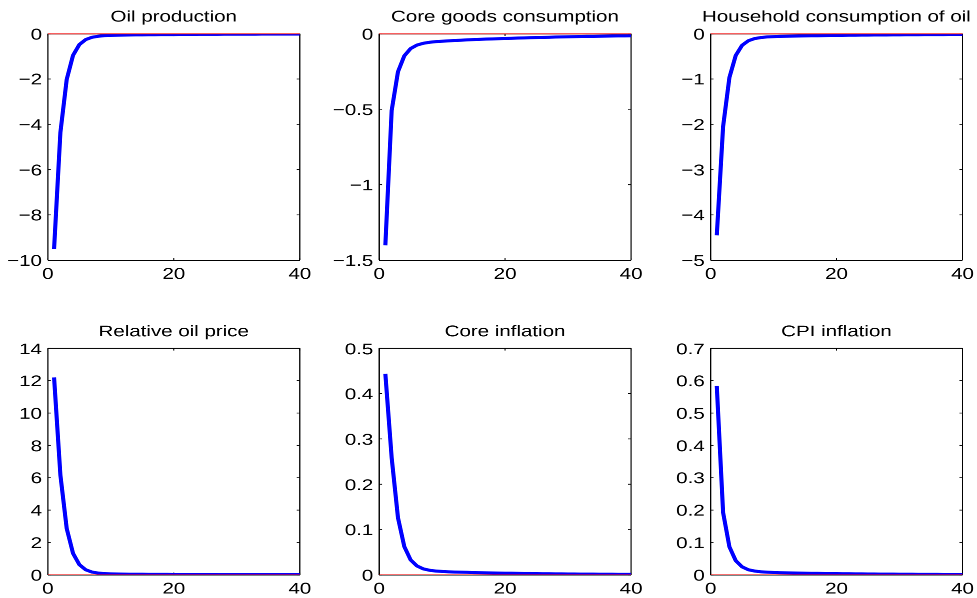


Figure 7. **Impulse response functions to a positive short-run productivity shock (ε_t^c) in the core sector.** This figure plots impulse response functions of oil production, household consumption of core goods, household consumption of oil, relative oil prices (P_t^O/P_t^C), core inflation, and CPI inflation. The size of the shock (ε_t^c) is one standard deviation $\sigma_c = 1.2\%$. The y-axis represents the percentage deviation from the steady state.

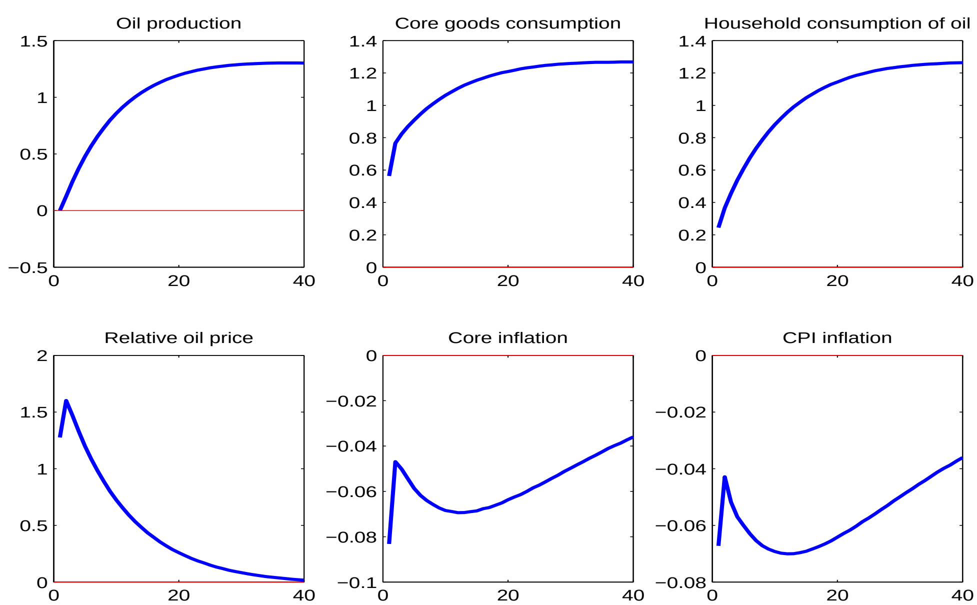


Figure 8. **Impulse response functions to a positive long-run productivity shock (ε_t^{xc}) in the core sector.** This figure plots impulse response functions of oil production, household consumption of core goods, household consumption of oil, relative oil prices (P_t^O/P_t^C), core inflation, and CPI inflation. The size of the shock (ε_t^{xc}) is one standard deviation $\sigma_{xc} = 0.07\%$. The y-axis represents the percentage deviation from the steady state.

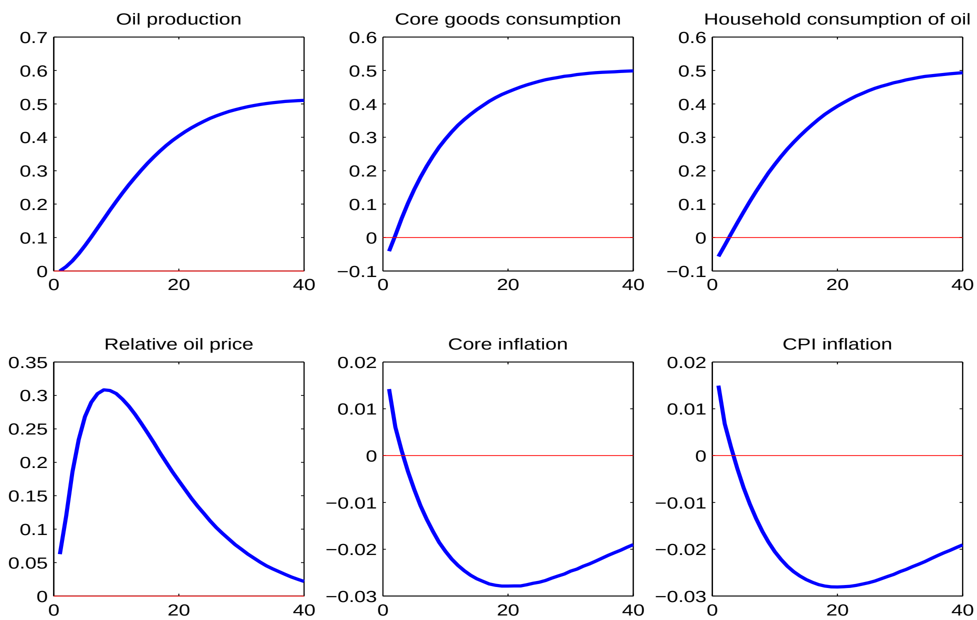


Figure 9. **Impulse response functions of 10-year real yields to the three productivity shocks.** The size of each shock is one standard deviation. The y-axis represents the percentage deviation from the steady state.

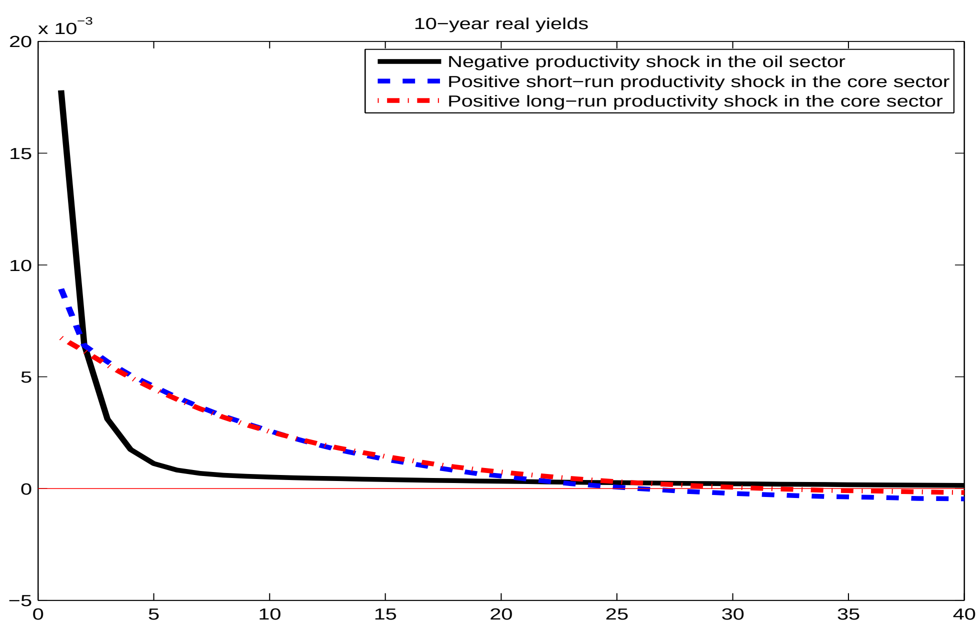


Figure 10. **Impulse response functions of 10-year breakeven inflation to the three productivity shocks.** The size of each shock is one standard deviation. The y-axis represents the percentage deviation from the steady state.

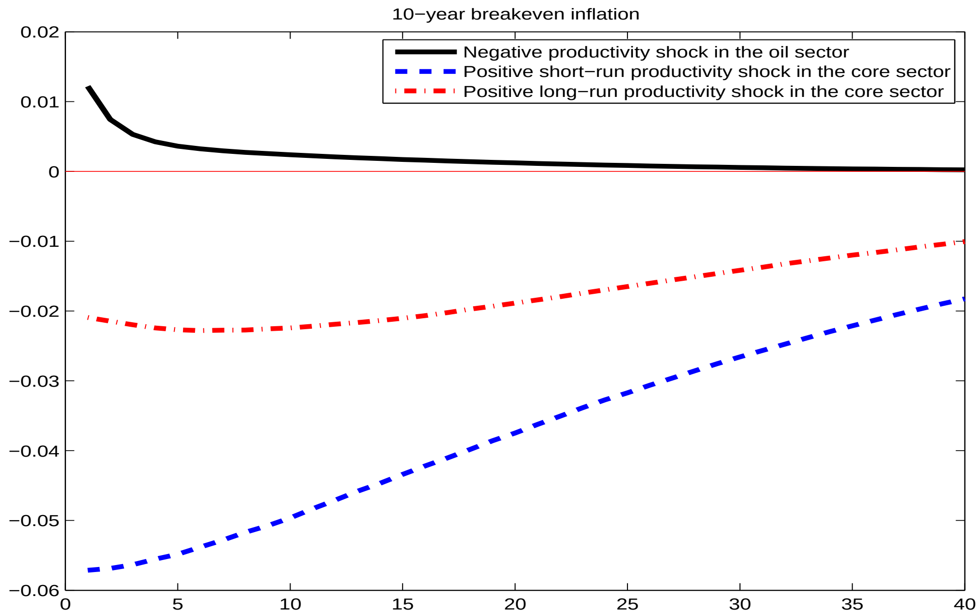


Figure 11. **Impulse response functions of 10-year nominal yields to the three productivity shocks.** The size of each shock is one standard deviation. The y-axis represents the percentage deviation from the steady state.

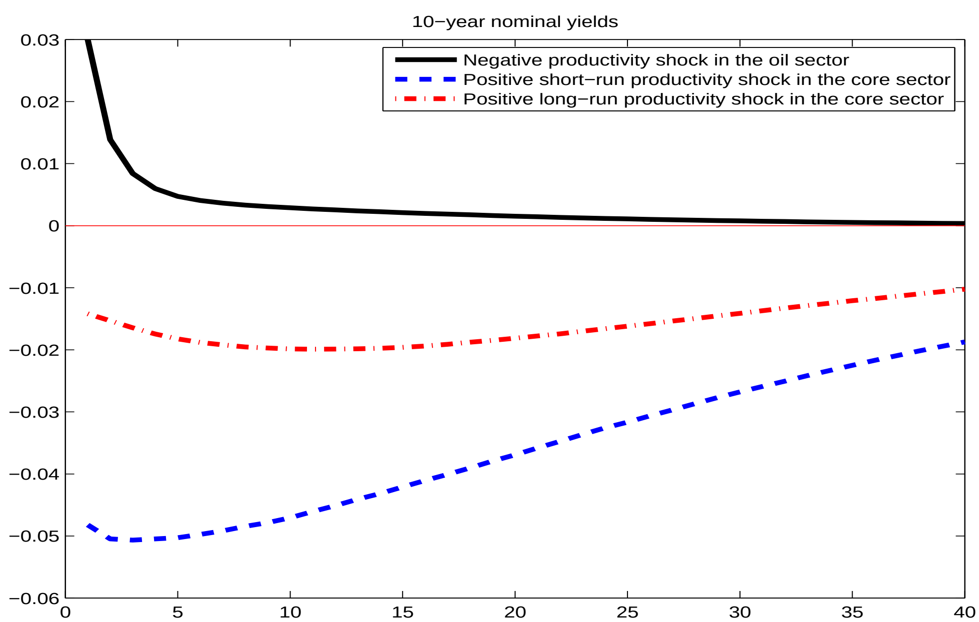


Figure 12. **Crude Oil Prices and U.S. 10-Year Breakeven Inflation.** This figure plots the crude oil prices (WTI-Cushing, Oklahoma) on the left y-axis, and U.S. 10-Year breakeven inflation on the right y-axis.

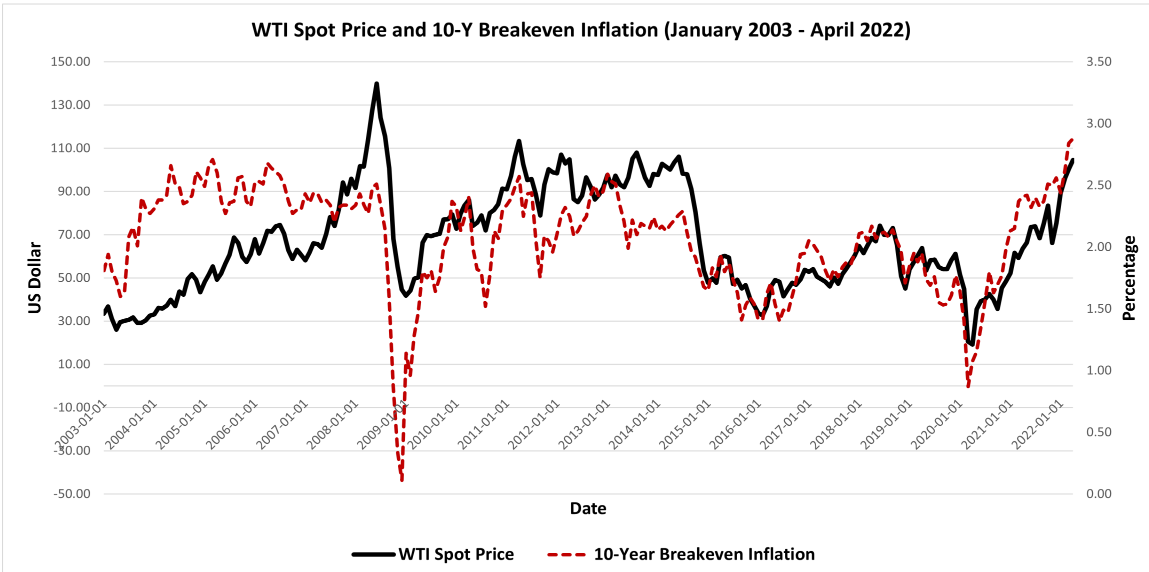
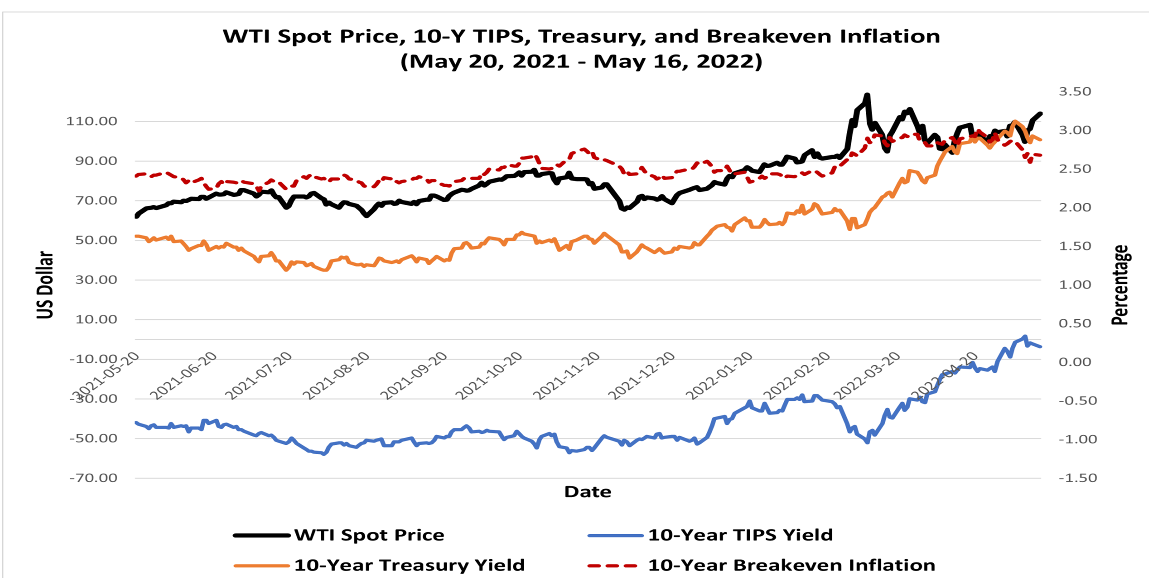


Figure 13. **Crude Oil Prices, U.S. 10-Year Treasury Yields, U.S. 10-Year TIPS Yields, and U.S. 10-Year Breakeven Inflation.** This figure plots the crude oil prices (WTI-Cushing, Oklahoma) on the left y-axis, and U.S. 10-Year Treasury yields, TIPS yields, and breakeven inflation on the right y-axis.



Appendix A Equilibrium conditions

A.1 Households

The Lagrangian for the household's problem is

$$\mathcal{L}^{HH} = V_0 + \sum_{t=0}^{\infty} \mu_t \left\{ (1-\beta) U_t^{1-\rho} + \beta \left(E_t V_{t+1}^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}} - V_t \right\} + \sum_{t=0}^{\infty} \lambda_t \left\{ R_{t-1} B_{t-1} + W_t N_t + D_t^C + D_t^O - (1-\xi) C_t P_t^C - \xi O_t^H P_t^O - B_t \right\}. \tag{A.1}$$

First order conditions with respect to choice variables C_t, O_t^H, N_t, and B_t give rise to the following equations

$$\frac{\phi \kappa_t N_t^{\nu}}{X_t^{1/\eta-\rho} C_t^{-1/\eta}} = \frac{W_t}{P_t^C}, \tag{A.2}$$

$$\left(\frac{O_t^H}{C_t} \right)^{-1/\eta} = \frac{P_t^O}{P_t^C}, \tag{A.3}$$

$$1 = E_t \beta \left(\frac{X_{t+1}}{X_t} \right)^{\frac{1}{\eta}-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\eta}} \left(\frac{V_{t+1}^{1-\rho}}{\left(E_t V_{t+1}^{(1-\gamma)/(1-\rho)} \right)^{1/(1-\gamma)}} \right)^{\rho-\gamma} \frac{P_t^C}{P_{t+1}^C} R_t. \tag{A.4}$$

Equation (A.2) represents the intratemporal relationship between consumption of core goods and labor supply. Equation (A.3) describes the intratemporal substitution between oil and consumption of core goods. Equation (A.4) is the Euler equation for consumption of core goods.

The nominal stochastic discount factor (SDF) $M_{t,t+1}^{\$}$ is defined as

$$M_{t,t+1}^{\$} \equiv \beta \left(\frac{X_{t+1}}{X_t} \right)^{\frac{1}{\eta}-\rho} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\eta}} \left(\frac{V_{t+1}^{1-\rho}}{\left(E_t V_{t+1}^{(1-\gamma)/(1-\rho)} \right)^{1/(1-\gamma)}} \right)^{\rho-\gamma} \frac{P_t^C}{P_{t+1}^C}. \tag{A.5}$$

The real stochastic discount factor (SDF) $M_{t,t+1}^{R,C}$, in the unit of core goods, could be defined as

$$M^{R,C}_{t,t+1} = M^{\$}_{t,t+1}\frac{P^C_{t+1}}{P^C_t}. \tag{A.6}$$

Alternatively, the real SDF can also be expressed in the unit of the consumption bundle

$$M^{R,X}_{t,t+1} \equiv M^{\$}_{t,t+1}\frac{P^X_{t+1}}{P^X_t}. \tag{A.7}$$

A.2 The oil firm

The Lagrangian for the oil firm's problem is

$$\mathcal{L}^O = E_t\sum_{j=0}^{\infty} M^{\$}_{t,t+j}\{Z^O_{t+j}K^O_{t+j-1}P^O_{t+j} - I^O_{t+j}P^C_{t+j} + q^o_{t+j}[(1-\delta^o)K^O_{t+j-1} + \Upsilon^O(I^O_{t+j}, K^O_{t+j-1}) - K^O_{t+j}]\}, \tag{A.8}$$

where the Lagrangian multiplier q^o_t is the shadow value of the capital (i.e., the Tobin's q).

The first order condition with respect to I^O_t is

$$P^C_t = q^o_t\Phi^O_I(I^O_t, K^O_{t-1})K^O_{t-1}, \tag{A.9}$$

where Φ^O_I is the partial derivative of Φ^O with respect to I^O_t.

The first order condition with respect to K^O_t is

$$q^o_t = E_t M^{\$}_{t,t+1}\left\{F^O_K(Z^O_{t+1}, K^O_t)P^O_{t+1} + q^o_{t+1}[(1-\delta^o) + \Phi^O_K(I^O_{t+1}, K^O_t)K^O_t + \Phi^O(I^O_{t+1}, K^O_t)]\right\}, \tag{A.10}$$

where $F^O_K(Z^O_{t+1}, K_t) \equiv Z^O_{t+1}$ and Φ^O_K is the partial derivative of Φ^O with respect to K^O_t.

A.3 The final goods firm

The first order condition of the final firm's problem is given in equation (17).

A.4 Intermediate goods firms

The Lagrangian for the intermediate goods firm's problem is

$$\mathcal{L}^C = E_t \sum_{j=0}^{\infty} M^{\$}_{t,t+j} \{ [P^C_{t+j}(i) Y^C_{t+j}(i) - \Psi_{t+j}(Y^C_{t+j}(i)) - P^C_{t+j} \frac{\vartheta}{2} \left(\frac{P^C_{t+j}(i)}{\bar{\pi} P^C_{t+j-1}(i)} - 1 \right)^2 Y^C_{t+j}$$
$$- P^C_{t+j} I^C_{t+j}(i)] + q^c_{t+j} [(1-\delta^c) K^C_{t+j-1} + \Upsilon^C(I^C_{t+j}, K^C_{t+j-1}) - K^C_{t+j}] \}, \tag{A.11}$$

where the Lagrangian multiplier q^c_t is the shadow value of the capital (i.e., the Tobin's q).

The first order condition with respect to I^C_t is

$$P^C_t = q^c_t \Phi^C_I (I^C_t(i), K^C_{t-1}(i)) K^C_{t-1}(i), \tag{A.12}$$

where Φ^C_I is the partial derivative of Φ^C with respect to I^C_t.

The first order condition with respect to K^C_t is

$$q^c_t = E_t M^{\$}_{t,t+1}$$
$$\left\{ F^C_K(Z^C_{t+1}, K^C_t(i)) P^C_{t+1}(i) + q^c_{t+1} [(1-\delta^c) + \Phi^C_K(I^C_{t+1}(i), K^C_t(i)) K^C_t(i) + \Phi^C(I^C_{t+1}(i), K^C_t(i))] \right\}, \tag{A.13}$$

where $F^C_K(Z^C_{t+1}, K^C_t(i))$ is the marginal productivity of capital and Φ^C_K is the partial derivative of Φ^C with respect to K^C_t.

The first order condition with respect to $P^C_t(i)$ is

$$P^C_t Y^C_t \left[(1-\varepsilon) \left(\frac{P^C_t(i)}{P^C_t} \right)^{-\varepsilon} \frac{1}{P^C_t} + \psi_t \varepsilon \left(\frac{P^C_t(i)}{P^C_t} \right)^{-\varepsilon-1} \frac{1}{(P^C_t)^2} - \vartheta \left(\frac{P^C_t(i)}{\bar{\pi} P^C_{t-1}(i)} - 1 \right) \frac{1}{\bar{\pi} P^C_{t-1}(i)} \right]$$
$$+ M^{\$}_{t,t+1} P^C_{t+1} Y^C_{t+1} \vartheta \left(\frac{P^C_{t+1}(i)}{\bar{\pi} P^C_t(i)} - 1 \right) \frac{P^C_{t+1}(i)}{\bar{\pi} (P^C_t(i))^2} = 0, \tag{A.14}$$

where ψ_t is the marginal cost defined in equation (A.18).

In a symmetric equilibrium, equation (A.14) is rewritten as

$$\vartheta \left(\frac{\pi_t^C}{\overline{\pi}} - 1 \right) \frac{\pi_t^C}{\overline{\pi}} = (1-\varepsilon) + \varepsilon \hat{\psi}_t + \vartheta E_t \left\{ M_{t,t+1}^{R,C} \left(\frac{\pi_{t+1}^C}{\overline{\pi}} - 1 \right) \frac{\pi_{t+1}^C}{\overline{\pi}} \frac{Y_{t+1}^C}{Y_t^C} \right\}, \tag{A.15}$$

where $\hat{\psi}_t \equiv \psi_t / P_t^C$ is the real marginal cost and $M_{t,t+1}$ is the real SDF defined in equation (A.6).

The first order condition of the cost minimization problem for a given level of output $Y_t^C(i)$ is

$$\frac{\alpha O_t^I(i)}{(1-\alpha-\omega) N_t(i)} = \frac{W_t}{P_t^O}. \tag{A.16}$$

Minimized cost function for a given level of output $Y_t^C(i)$ is

$$\Psi(Y_t^C(i)) = (1-\omega)\alpha^{-\frac{\alpha}{1-\omega}}(1-\alpha-\omega)^{-\frac{1-\alpha-\omega}{1-\omega}}(Z_t^C)^{-\frac{\alpha}{1-\omega}}(K_{t-1}^C(i))^{-\frac{\omega}{1-\omega}}(W_t)^{\frac{\alpha}{1-\omega}}(P_t^O)^{\frac{1-\alpha-\omega}{1-\omega}}(Y_t^C(i))^{\frac{1}{1-\omega}}. \tag{A.17}$$

Marginal cost function for a given level of output $Y_t^C(i)$ is

$$\psi(Y_t^C(i)) \equiv \Psi'(Y_t^C(i)) = \alpha^{-\frac{\alpha}{1-\omega}}(1-\alpha-\omega)^{-\frac{1-\alpha-\omega}{1-\omega}}(Z_t^C)^{-\frac{\alpha}{1-\omega}}(K_{t-1}^C(i))^{-\frac{\omega}{1-\omega}}(W_t)^{\frac{\alpha}{1-\omega}}(P_t^O)^{\frac{1-\alpha-\omega}{1-\omega}}(Y_t^C(i))^{\frac{\omega}{1-\omega}}. \tag{A.18}$$

A.5 Market clearing conditions

In equilibrium, all markets are clear. The aggregate oil resource constraint is

$$O_t^H + O_t^I = Y_t^O. \tag{A.19}$$

In the symmetric equilibrium, the aggregate resource constraint of final consumption goods becomes

$$C_t + I_t^O + I_t^C = \left(1 - \frac{\vartheta}{2}\left(\frac{\pi_t^C}{\overline{\pi}} - 1\right)^2\right) Y_t^C, \tag{A.20}$$

where $\pi_t^C = P_t^C / P_{t-1}^C$.